JAN 15

REPORT DOCUMENTATION PAGE

AFRL-SR-BL-TR-02-

0049

1. REPORT DATE *(DD-MM-YYYY)* 27-12-2001	**2. REPORT TYPE** Final	**3. DATES COVERED** *(From - To)* Feb.1,1999-Sept. 30,2001
4. TITLE AND SUBTITLE Optimization of Subsystem Integration via the 2nd Law of Thermodynamics		**5a. CONTRACT NUMBER** F49620-99-0107
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) Richard A. Gaggioli David M. Paulus, Jr.		**5d. PROJECT NUMBER**
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Marquette University Dept. of Mech. & Ind. Eng. 1515 West Wisconsin Avenue Milwaukee, Wisconsin 53233		**8. PERFORMING ORGANIZATION REPORT NUMBER** MUCES-AFOSR-02-01
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Air Force Office of Scientific Research 801 N. Randolph St. Suite 732 Arlington, VA 22203-1977		**10. SPONSOR/MONITOR'S ACRONYM(S)** AFOSR
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

Dr. Steven Walker (Program Manager) Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Engineering design of a product (such as an aircraft) invariably seeks to achieve several desiderata - several features and performances desired by the customer (actual or perceived customer). Usually the desiderata are conflicting, in the sense that improvement of one requires some sacrifice of another. Therefore, design requires that a means be employed for "balancing" the various desiderata, in order to determine and then account for their relative importance. Optimal design seeks to provide such a means, by establishing a mathematical "objective function" with a measure of *value* (to the customer) as the dependent variable and the desiderata as independent variables. A procedure for establishing such an objective function is presented in this report, along with a rational methodology for "subsystem integration" - a procedure for establishing subsystem objective functions such that, when each is individually optimized, the overall objective function is optimized.

15. SUBJECT TERMS

Multiple Objective Optimization, Subsystem Integration, Exergy, Thermoeconomics

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** R.A. Gaggioli
a. REPORT Unclassified	**b. ABSTRACT** Unclassified	**c. THIS PAGE** Unclassified		74	**19b. TELEPHONE NUMBER** *(include area code)* 414-288-7316

Final Report
on
OPTIMIZATION OF SUBSYSTEM INTEGRATION VIA THE 2ND LAW OF THERMODYNAMICS

to

Air Force Office of Scientific Research

from

R.A. Gaggioli
Marquette University
27 December 2001

EXECUTIVE SUMMARY

Optimal Design

Engineering design of a product (such as an aircraft) invariably seeks to achieve several desiderata – several features and performances desired by the customer (actual or perceived customer). Here, a "feature" means a characteristic that the product either will have or won't have (such as the capability to land on a carrier), while a "performance" indicates an amount of a characteristic that the product must have (such as top speed).

Usually the desiderata are conflicting, in the sense that improvement of one requires some sacrifice of another. For example, one typical desideratum of a vehicle is low cost and another is high fuel economy. But improvement of fuel economy entails greater cost.

Therefore, design requires that a means be employed for "balancing" the various desiderata, in order to determine and then account for their relative importance. Traditionally, this has been accomplished, implicitly if not explicitly, by "jawboning" – with real or hypothetical advocates for each desideratum arguing for theirs, before a "judge" or a "court" that decides. Such a procedure has been effective in the past, even though it entails much *subjectivity* and, consequently, takes a lot of *time*. So, a *rational* means is needed for taking the various desiderata into account.

Optimal design seeks to provide such a means. The key is to establish a mathematical "objective function" with a measure of *value* (to the customer) as the dependent variable and the desiderata as independent variables. A common measure of value is money ("dollar-value"). However, in the case of military aircraft – where the total budget for the "fleet" to be produced is dictated, not by the customer (say the Department of Defense), but by a "third party" (Congress) – the measure of value might well be taken to be the size of the fleet (i.e., the number of aircraft to be produced, the "production").

More difficult than the selection of the measure for value is the establishment of the function – that is, the relationship that determines the influence upon value of (a) the existence of a feature, and (b) amounts of each performance.

This report, within the context of a general outline for thermal design, introduces a procedure for establishing an objective function, rationally accounting for each desideratum, weighting their relative importance and, in the case of a performance, quantifying the value of varying amounts thereof. This procedure is illustrated by an elementary example, applicable to certain choices made in the design of a light aircraft. In particular, to design decisions regarding *selections* – choices from among alternatives (which differ in performance or features) – for an engine, a generator, and a propeller.

Subsystem Decomposition

When designing a system such as an aircraft, the design encompasses several subsystems, such as engine, propeller, generator, ... For a *simple* system, like a *light* aircraft, the decisions regarding individual subsystems can be guided by direct application of the

overall, system objective function. That is what was done in the example referred to above.

When designing a *complex* system such as a *combat* aircraft, in practice there is the need for "concurrent engineering." The individual designs of different subsystems are carried out concurrently by distinct engineers (or teams) – each with their own expertise. Unfortunately, independent optimization of the subsystems, separately, in no way insures optimal design of the overall, complex system.

Again, traditionally, the overall "optimization" has been carried out subjectively, depending upon the experience and judgment of a chief "project" engineer to make decisions regarding parameters that reflect subsystem interactions. Here too, "jawboning" has been an influence, with each team leader advocating parameter values that would be advantageous to the team's subsystem.

A procedure that would streamline the decision-making and make it objective, is called for. Such a method would need to decompose the overall objective function into individual objective functions for the subsystems, in a manner such that somehow optimization of the subsystem objectives would assure overall optimization.

This report also presents a rational methodology for "subsystem integration." – a procedure for establishing subsystem objective functions such that, when each is individually optimized, the overall objective function is optimized. For each subsystem objective, the procedure deduces weighting and cost factors from the basic, overall objective function. This methodology is also illustrated by application to the aforementioned light aircraft. The application of the methodology requires a means for assigning costs to any "energy" flows to and from subsystems, to be discussed next.

Thermoeconomic Costing

In the overall objective for an aircraft, fuel consumption is invariably a factor; one desideratum is always to reduce fuel consumption. In one way or another, every subsystem consumes "energy" – that is, every subsystem is a cause of fuel consumption. Some aircraft subsystems use energy only passively, energy required for "lift", to keep the subsystem aloft, and energy required for acceleration. Other subsystems actively consume energy, in one form or another, and many of these also deliver energy, in a different form. The design objective function for any subsystem needs to debit the subsystem for every unit of energy consumed and give a credit for any energy delivered. The value of a unit of energy, say a kilowatt-hour, depends upon how much fuel has been consumed to generate the unit and upon other, associated investments, such as the cost of all the equipment employed. In order to follow all the costs through the complex, overall aircraft, and assign unit values at different junctures, the proper commodity – the proper "energy" to be tracked – is not *textbook* energy, but the concept that, today, is called *exergy*. Exergy (also called "availability") is the appropriate measure of *marketplace* energy – that is, of "potential to drive processes." It is the key not only for proper *cost accounting* of "energy" but also, even beforehand, to *analysis* – i.e., to pinpointing and

quantification of inefficiencies in energy-conversion systems. (And, basically, an aircraft is one example of an energy-conversion system.)

In this report, the overall objective function for a light aircraft is decomposed into subsystem objective functions, and exergy costing is then employed to account for debits and credits associated with "energy" flows. Each of the subsystems is then optimized independently. And, it is shown that the subsystem optima that are achieved lead to the same decisions as those made without decomposition – the same as the decisions reached by applying the overall objective function alone. This comparison, possible because of the simple subsystem applications within the context of a relatively simple overall system – the light aircraft – provides evidence that backs up (a) the procedure for establishing the overall objective function, (b) the procedure for subsystem composition, (c) the thermoeconomic methods of exergy costing, and (d) the means developed to account for "the exergy of lift".

Exergy of Lift

Clearly, the weight of any subsystem penalizes the performance of an aircraft, increasing the fuel consumption and decreasing the acceleration. In order to employ exergy costing to achieve subsystem decomposition, it was evident that a means was needed to account for weight. This led to a new theoretical development, the deduction of a new component of exergy, needed for exergetic analysis and costing in the context of aircraft.

Summary of Results Obtained

1. A procedure for the development of an overall objective function for *rational* optimal design of an aircraft, taking into account all of the desiderata and their relative importance.
2. A procedure for "concurrent engineering" of subsystems – "subsystem integration" – that develops an objective function for each energy-conversion subsystem (using the overall objective function to derive weighting factors employed in the subsystem objectives) so that subsystem optimization leads to overall aircraft optimization.
3. The application of exergetic costing – "thermoeconomics" – in order to properly assign the impact on fuel consumption in each of the subsystem objective functions (i.e., the costs of fuel, of energy-conversion equipment capital, of energy-conversion equipment inefficiencies, of energy-conversion equipment weight).
4. The development of theoretical relationships for evaluating a new component of exergy – "exergy of lift" – needed to properly account for and track the exergy costs (i.e., the costs associated with energy-conversion equipment weight).

Together, the foregoing developments establish an overall, rational procedure for vehicle design. The results are developed in detail in the following report and, more concisely, in the technical papers appended to the report.

Recommendations

- *General.* It is suggested that, in order to further test and perfect them,
    - The methods, which so far have been applied only to the *selection* of subsystems, be applied to the detailed design of the subsystems.
    - The methods, which so far have been applied only to the simple subsystems of a simple aircraft, should be applied to the design of a complete aircraft – say a relatively simple one such as an Unmanned Reconnaissance Vehicle – including all of its subsystems.

- *Specific.* It is suggested that the following refinements be pursued.
    - During the development of the overall objective function, more sophisticated relations might well be employed for representation of the importance of the various desiderata.
    - During the development of the overall objective function, the "interaction" of desiderata could be taken into account. That is the "representations of importance" could include more than one independent variable, more than one desideratum.
    - During the application of exergetic costing, the determination of marginal costs in lieu of average costs needs to be pursued further.

TABLE OF CONTENTS

Final Report
on
OPTIMIZATION OF SUBSYSTEM INTEGRATION VIA THE 2ND LAW OF THERMODYNAMICS

to

Air Force Office of Scientific Research

from

R.A. Gaggioli
Marquette University
27 December 2001

I. NOMENCLATURE

Variable	Use
a	aspect ratio
A	area
C	heat capacity
c	cost
C_D, C_L	coefficient of drag, lift
E	energy
F	fuel
$\mathbf{F}$	force
g	acceleration of gravity
J	objective function
m	mass
P	performance
R	range
S,s	entropy, specific entropy, sensitivity coefficient
T	time factor, temperature
v	value
V	speed
$\mathbf{V}$	velocity (scalar)
W	weighting factor
X,x	exergy, specific exergy
y	decision variable
Z	capital

Table 1: Variables

Greek Variable	Use
Π	profit
μ	viscosity
ρ	density

Table 2: Greek Variables

Subscript	Use
DF	duct firing
$HRSG$	heat recovery steam generator
m	mass
δ	destruction
π	production

Table 3: Subscripts

II. MOTIVATION

This work was performed with Air Force Office of Research Support funding for the optimization of aircraft energy conversion subsystems. The optimization of these subsystems proves to be more intricate than many traditional energy system design problems, as figures of merit for the vehicle as a whole are difficult to translate into values that will be meaningful to an on-board energy system (or subsystem or device). These figures of merit are manifold for a modern fighter, as it must excel in several competing performance areas. One objective is to have a low signature, i.e., be stealthy. Another objective is rapid acceleration, typically measured from Mach 0.8 to Mach 1.2. The aircraft should have a long range and a great payload.

Here these problems will be addressed in two manners. First, an overall design methodology, from the feasibility study through detailed design, for thermal systems will be developed in a general fashion. Secondly, the detailed design phase will be covered in detail. In order to support these two areas, a means of developing an overall objective function for a vehicle with multiple objectives will be given. Additionally, it will be shown how to develop subsidiary objective functions for systems, subsystems and devices from this overall function.

Although particular attention will be paid to vehicular energy systems, namely those for aircraft, the general approach for thermal system design outlined here will be as valid for a stationary plant as for a vehicle. The approach will be bounded in the following manner. It is assumed that a need for an energy system has been given. Only thermal design will be considered explicitly. Maintainability, reliability, architecture and mechanical design are beyond the scope of this work. Whenever possible, the overall design processes as laid out in Ostrofsky, 1977 and Woodson, 1966 are followed. This is to allow the thermal design process here to share the same vocabulary and jargon with general design as a standard. In many instances, however, the steps and substeps given in these are not are not applicable, or out of place, for the thermal design process. In such cases they are not used, or occur at different times.

For the design process to occur in a rational, quantifiable fashion, a mathematical objective function is necessary. Current practice in vehicular design is to list all of the desired objectives (e.g., speed, payload, climb rate, acceleration, etc.) and then to express some of them in quantitative terms and some solely in qualitative terms. But there is not an objective procedure for weighting the relative importance of these often-competing desiderata. The "weighting" is carried out, subjectively, during the course of the design process, primarily as a consequence of conversations between different involved personnel -- such as the customer, the end-users, the chief engineer, the various specialized design teams, etc.

While such conversations will always be important, in order to expedite the design process and make it more efficient, it is desirable to establish a rational objective function for each overall vehicle project, including a quantitative means for assessing the importance of the various desired objectives.

Extensive work has been done in the area of optimized design of energy systems that have an end purpose of providing a mass, heat or power flow. Examples include the "Thermoeconomics" of El-Sayed and co-workers (e.g., El-Sayed and Evans, 1970, Evans and von Spakovsky, 1984). However, many energy systems are used in applications for which the end product is not energy, such as vehicular applications. In such instances the energy systems impact not only lifetime costs of the application, but also its performance. It is necessary to take these performance impacts into account in order properly to optimize these energy systems. Indeed, for some vehicles, such as a fighter or a Formula-1 automobile, performance **is** the overwhelming objective. Often these performance desiderata are competing, as per the example of the fighter aircraft above. The determination of the optimum balance between these desiderata is naturally a greater challenge than many traditional, stationary energy system design problems.

Detailed design and optimization will be covered in depth. It is in this phase that second law techniques enable a design team to approach an optimal design while concurrently performing detailed design and selection of components to an energy system. The application of the second law of thermodynamics, combined with economics, allows for decomposition of systems into devices and subsystems that may be independently optimized. This theory has been developed and validated by El-Sayed and coworkers. (See for example, El-Sayed and Evans, 1970; El-Sayed, 1995.)

Additional second law and thermoeconomic considerations will be developed as necessary to support the design of vehicular energy systems. These new developments include the exergy of lift, the flow of exergy (and its cost) in an aircraft, the reference environment and marginal exergy costing. (Naturally the latter two areas have much broader applications than solely vehicle design.)

To illustrate the concepts of this report, an overall objective function for a light aircraft will be formulated. The exergy flow in this aircraft will be diagrammed for several realms of flight. Then cost balances will be applied to find the marginal costs per unit exergy. Finally, an engine and an alternator will be selected for this aircraft using this information, for optimal design of the aircraft.

III. DEFINITIONS

(a) System

An energy conversion system is a device or set of devices employed to produce energy products (i.e., exergy in some form) at the expense of the consumption of fuel (exergy in a different form. Here the word *system* is used for the largest grouping of equipment with pure exergy inputs and outputs. One example: A combined cycle power plant. *For aircraft*, a system will be considered a collection of devices used to provide a fundamental requirement for flight. An aircraft example of a system would be the propulsion system.

(b) Subsystem

An energy conversion subsystem is a set of devices, contained within a system, configured together to produce products used within a system. The realms of the system and subsystem are not clear-cut; however, the subsystem's products and fuels are dependent on the requirements of the overall system. Examples in a combined cycle power plant: the gas turbine engine and the heat recovery steam generator (HRSG). *For aircraft*, this report will define an energy conversion subsystem as an energy conversion system not directly involved in the production of thrust. One aircraft example is an environmental control system.

(c) Device

An energy conversion device is defined here as hardware, normally contained in a single housing, designed to produce a single product with only one or two fuels. Examples in the combined cycle power plant: within the gas turbine, the compressor, combustion chamber(s) and the turbine; within the HRSG the various heat exchangers; in the remainder, pumps and steam turbines. Examples in an aircraft: the compressor, a hydraulic pump, an alternator…

(d) System-level Decision Variable

A system level decision variable is a variable, that, when changed, will greatly affect the (optimal) design and/or selection of subsystems and devices. In the combined cycle power plant system level variables may include gas turbine pressure ratio, firing temperature and steam side operating pressures.

(e) Device-level Decision Variable

A device level variable, when perturbed, has little or no effects on optimal values of system level or other device's decision variables. One example of a device decision variable in a combined cycle power plant would be the peak efficiency of a HRSG feed pump.

(f) Fuel

Any exergy input to a system or subsystem is considered a fuel. This is beyond the traditional thought of fuel being something "burned" as an energy source. Of course, the exergy source for any device is at least partially "burned" or depleted driving whatever process occurs in the device.

(g) Product

The desired exergy outputs of an energy system or subsystem are products. The product of a vehicle is performance: speed, acceleration, payload, etc.

(h) Byproduct

A byproduct is an exergy output of a system that is not necessarily desirable but unavoidable due to the process.

(i) Candidate System

A candidate system is a system that satisfies the laws of physics and the demands of the problem.

(j) Constraint

A constraint limits the designer's freedom in conceiving a design. These constraints may be on the design itself, e.g. the design must fit in a certain place, or on the design process. A typical constraint on the design process itself is time.

IV. A GENERAL OUTLINE OF THE DESIGN PROCESS FOR THERMAL SYSTEMS

As presented here, the design of thermal systems has three phases. In the first, the *feasibility study*, a set of possible solutions, consisting of candidate systems, is developed. In the

next phase, *preliminary design*, the best design of the candidate systems is chosen for development. Finally, during *detailed design* all hardware is selected and decision variable concretized.

(a) Feasibility Study

According to Ostrofsky, "The purpose of the feasibility study is to develop a set of useful solutions to meet the needs... The primary importance of the feasibility study is that it is the foundation for all that occurs subsequently in the design-plan for the system."[1]

Applied to thermal design, the feasibility study should generate a set of solutions, all of which meet the design needs, that (a) satisfy the laws of thermodynamics and (b) are technologically achievable within any time constraints that exist. This is achieved through (i) a product, fuel and byproduct analysis which yields (ii) a problem formulation. After these are complete, a set of possible solutions are proposed, and screened so that they meet the two conditions stated above.

(i) Product, Fuel and Byproduct Analysis

Before any solutions may be conceived, the energy system designer must know (a) what the system is supposed to produce (the product(s)), (b) what is available for the system to consume (the fuel(s)) and (c) the possible byproducts (both undesirable and possibly desirable).

Typical products for an energy system include, but are not limited to, shaft power, heating or cooling loads and compressed air or other mass flows.

Fuels may be fuels in the conventional sense, such as fossil or nuclear fuels, or mass, heat or work streams from another system or subsystem.

At this stage in the design process, undesirable byproducts should be listed. These include, among others, emissions, noise, weight, volume and drag.

(ii) Problem Formulation

With the fuels, products and byproducts listed the problem statement may be crafted. In addition to stating the information generated from product, fuel and byproduct analysis, any constraints on the system must be included. Examples of such constraints are output (product) requirements and size, emissions and noise limitations. Additionally, time constraints on the design should be noted. These may have a large impact on the screening of the solutions generated in the next phase of the feasibility study.

(iii) Solution Generation

Solution generation involves the "brainstorming" of possible configurations and processes to meet the problem. At this stage, no consideration should be given to the viability of the brainstormed solutions. The end product of solution generation is a set of candidate systems.

(iv) Solution Screening

Solution screening involves the asking the following questions:

---

[1] Ostrofsky, 1977, pg. 29

1. Is the system physically attainable? That is, can the system meet the needs of the problem without violating the laws of physics? For energy systems, the candidate system must obey the first and second laws of thermodynamics.
2. Is the system capable of meeting all of the non-time constraints?
3. Can the system be realistically developed within the time constraints given? If new technology is involved or is to be developed, the probability of completion must be assessed. Is the risk with this technology acceptable, or does a backup to it exist?

In order for a candidate system to advance beyond screening, the answers to all of these questions must be yes.

Question 1 should be answered by creating a basic thermodynamic model of the candidate system. Equipment performance may be assumed to have some reasonable value, or assumed reversible. (It is to be remembered that any system that is **possible** should not be failed here.) Not only will such a model show that the candidate does not violate the laws of physics, but may also serve as a basis for optimization during preliminary activities.

(v) Scenarios

The following two scenarios will illustrate what is desired as a result of the feasibility study. Furthermore, both of these scenarios will reappear later as illustrations of points developed later.

1) A Light Aircraft Propulsion System

A propulsion system is needed for a light aircraft's airframe. The light aircraft, in this case, is the Glastar kit-built aircraft (see Figure 1 and Appendix A). The product, fuel and byproduct analysis is relatively straightforward: the product, of course, is thrust; the fuel may be automotive fuel, aviation fuel or jet fuel; byproducts include noise and vibration, as well as weight and drag.

One way of expressing the problem statement is thus:

> Develop a propulsion system for the given airframe. Cruise should be in the 140-knot range; good short-field performance should be achieved. Endurance should be in the 3-6 hour range. As aviation fuel, automotive fuel and jet fuel are commonly available at airports, all three are acceptable fuels.

The following lists of solutions is generated:

1. Piston engine with a constant-speed propeller
2. Piston engine with a fixed-pitch propeller
3. Diesel engine
4. Gas turbine engine (turboprop)
5. Jet engine (turbojet or turbofan)

Initial screening eliminates number 5 immediately, as the airframe is not adaptable to a jet engine. Likewise, numbers 3 and 4 will be eliminated here, as there currently are no suitable

turboprop installations for this airframe.[2] Thus, options 1 and 2 will advance to preliminary design.

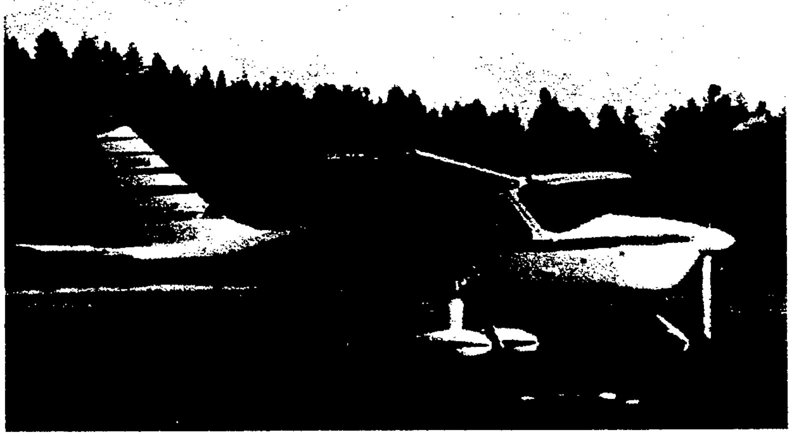


Figure 1: Glastar® Kit-built Aircraft

2) A "Beer and Soup" Plant[3]

What follows is a fictional problem statement for machinery at a fictional "beer and soup" plant:

> A plant is producing beer and soup. In the final processing, it is necessary to hold the soup at 368K and the beer at 282K. Typical ambient temperatures are between these two values. Natural gas and electricity are available at the plant sight as energy sources. 166 kW of heat energy must be supplied to the soup and 40 kW removed from the beer in the process.

One can easily imagine several solutions. The beer may be refrigerated, while the soup is heated by natural gas-fired burners. The soup could be heated by electric heating coils. The soup could be heated by steam generated in a gas-fired or electric boiler. Or, a heat-pump/refrigeration unit, as shown in Figure 2, could be employed. Finally, more than one of these methods could be combined.

---

[2]This is really a time constraint, as a turboprop or diesel engine *could* certainly be developed for an aircraft such as the Glastar. In fact, much research is under way in this area.

[3] Although here described as a "Beer and Soup" plant, such a system could also be employed on an aircraft for combined cooling and heating loads, such as avionics and cabin heat.

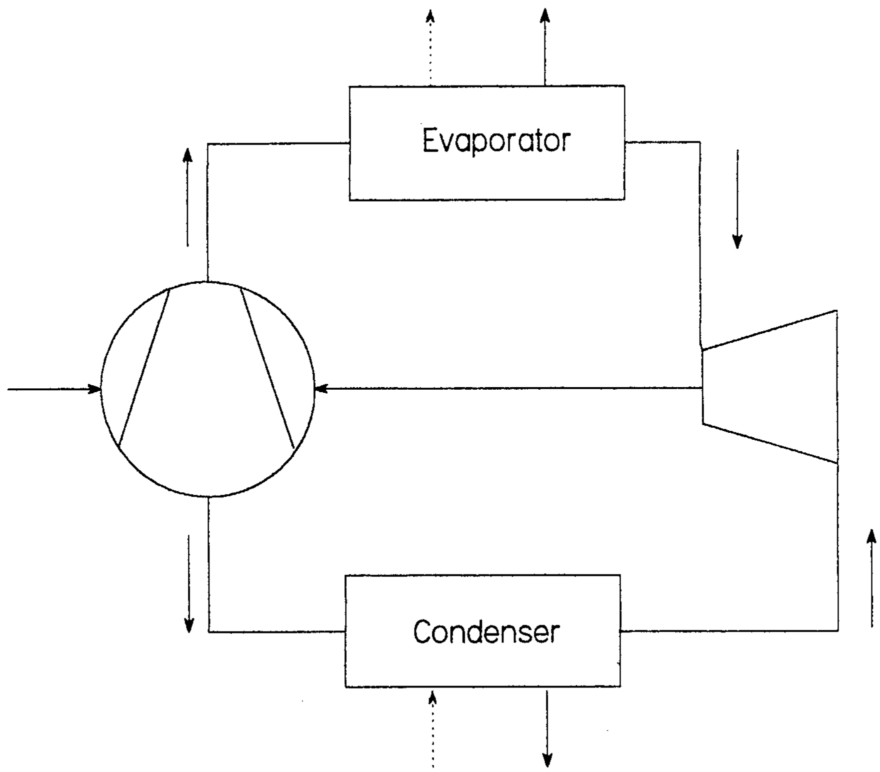


Figure 2: Heat Pump/Refrigeration System

(b) Preliminary Design

The end result of the preliminary activities should be the selection of the best configuration from all of the candidate systems. In order to accomplish this, an objective function must be formulated, basic optimization must be performed on the candidate systems and the advantages and disadvantages of each candidate system should be listed.

(i) The Objective Function

It is not possible to properly evaluate the candidate systems without a function that mathematically quantifies their relative merit. For many energy systems, especially stationary systems, the objective function is the lifecycle cost, and it is to be minimized. An alternative is a profit to be maximized.

When an energy system is used in a vehicle, either as the prime mover or for other purposes, such as climate control, the objective function becomes more complicated. This is because although energy drives a vehicle, the purpose of the vehicle is not simply to perform energy conversion. A vehicle exists to provide performance, and this cannot be expressed directly in energy terms. Therefore, the objective function for a vehicle must account for the benefits of performance. If an energy system is to be designed for use on-board a vehicle, the objectives for the overall vehicle must be translated into terms applicable to the energy system itself. Such translation involves thermoeconomics and weighting factors. Additional costs, outside of the typical fuel and capital costs, may need to be included. One example of an additional cost, highly applicable to aircraft design, is the cost of weight.

Qualitative objectives for Scenario 1, presented above, are listed in Table 4. The formulation of objective functions is discussed in detail in Section 4. Additionally, overall and subsystem objective functions are developed in detail during Section 7, in a continuation of Scenario 1.

Objective	Extremization
Cruise speed at 75% power	maximize
Range at 65% power	maximize
Rate of climb at gross weight	maximize
Take-off roll	minimize
Useful load	maximize
Cost	minimize

Table 4: Qualitative Objectives for Scenario 1.

(ii) Optimization of Candidate Systems and the Sensitivity Analysis

The purpose of the optimization of candidate systems is to compare candidate systems on an equal basis. Without this optimization, one may be comparing a poor iteration of one candidate to a good iteration of another, with the latter appearing to be the better candidate. After optimization of both, it could become clear that the initially poorer performing system is indeed the better choice.

The purpose of the sensitivity analysis is to evaluate the rate of change of the objective function to each decision variable of each candidate system. The sensitivity analysis tells the designer how critical it is to achieve the optimal values of each decision variable. If a decision variable has an optimal value that, for whatever reason, may be difficult to achieve, a low sensitivity to this value is naturally desirable.

During the course of optimization and the sensitivity analysis it will be necessary to determine which design variables are system-level and which are device-level. This information is important not only for later occurring detail activities, but also here, as in this stage of the design process it is desirable to optimize on system-level variables only, while, in this stage, typical values should be used for device-level variables. Generic models should be used for equipment and costing. The idea is to provide sufficient information for candidate screening; excess effort spent on the candidates not chosen is wasted.

Optimization requires an optimization algorithm, a simulation algorithm and cost models. These topics are well developed in the literature.[4]

Mathematically the sensitivity analysis is performed by taking the partial derivative of the objective function with respect to each system-level decision variable.

$$S_i = \frac{\partial J}{\partial y_i} \tag{1}$$

where S_i is the sensitivity coefficient for objective function J and decision variable y_i.

The sensitivity analysis and the optimizations are intricately intertwined. For the sensitivity analysis to be accurate, the values of the decision variables should be near their optimum values. If it is not clear which variables are system-level and which are device-level, the sensitivity analysis may need to be extended so as to investigate the effects of decision

[4] See, for example El-Sayed and Evans, 1970; El-Sayed, 1996

variables on one another. Namely, partial derivatives of the optimal value of a decision variable with respect to the variable in question may be taken.

$$s_{ij} = \frac{\partial y_{i,opt}}{\partial y_{j \neq i}} \tag{2}$$

These derivatives should be of relatively low magnitude for the variable in question to be considered device-level.

After optimization the value of the objective function, the value of all relevant decision variables and the sensitivity of the objective function to these should be listed for each candidate system.

(iii) Listing of Advantages and Disadvantages

Ideally, all advantages and disadvantages of the candidate systems will be mathematically included in the objective function. However, in reality this will not always be the case. Many desirable traits, such as maintainability and ease of replacement part procurement may be difficult to quantify. Any of these traits should be listed along with the values of the objective functions.

To continue with the example presented by scenario 1, Table 3.5 shows some possible piston engine combinations along with their advantages and disadvantages.

Engine-Propeller	Advantages	Disadvantages
Lycoming 0-320 or 0-360 engine[5], fixed-pitch propeller	Simplicity, less maintenance, engine parts are readily available, aircraft mechanics are very familiar with engine. Lightweight propeller.	Propeller will always compromise speed, climb or both.
Lycoming O-320 or O-360 engine, constant-speed propeller	Engine parts are readily available, aircraft mechanics are very familiar with engine. Very good all-round performance. Lower cruise fuel consumption.	Constant speed propeller will add complexity, weight and maintenance costs.
Walther-LOM 160 horsepower engine[6], constant speed propeller	Overhead-camshaft design is more efficient. Fuel injection eliminates manual leaning of mixture. Supercharger provides better power at altitude than the Lycoming O-320 does. Lightweight engine.	Constant speed propeller will add complexity and maintenance costs. LOM engine not nearly as common as Lycoming. The engine's configuration requires a longer cowling. Engine is only rated at 140 horsepower continuous.

Table 5: Engine-propeller combination advantages and disadvantages

[5] The O here stands for opposed, and the number for the engine displacement. These engines are carbureted, produce from 150-180 horsepower, are built in the United States and are very standard in light aircraft.

[6] The LOM engine is a fuel injected and supercharged inline engine.

(iv) Choosing the Best Candidate

The best candidate will normally have the highest value of the objective function. However, before simply choosing the candidate with the highest value, the following questions should be answered. (This list should not be construed as all-inclusive.)

1. Do any of the decision variables have values, after optimization, that may be difficult to achieve due to technological difficulties or constraints. Until detail design is complete, there is always some doubt as to whether the system can be built as envisioned during this phase. The effect on the value of the objective function, which occurs due to movement from the optimal value, may be estimated from the results of the sensitivity analysis with

$$\Delta J \approx S_i \Delta y_i \tag{3}$$

   With this, the effects of non-achievement of the results of the optimization may be estimated, and the risk associated with developing a candidate may be evaluated.

2. Does the candidate with the highest value of the objective function have any major (non-quantified) disadvantages to the other candidates?
3. Is there uncertainty in meeting time constraints?

Yes answers to any questions like these may be valid reasons for choosing a candidate other than that with the highest value of the objective function. Nonetheless, with a good objective function, a cost may be put on a choice. For example, one might say choosing design X over Y will cost so much, but will yield a greater level of maintainability. Likewise, risk-benefit analysis may be accomplished.

If multiple candidates have similar numerical values for the overall objective, additional effort is necessary to make the selection. It may be necessary to perform a more detailed optimization to confidently choose one candidate system. Also, for similar values in the objective functions, generally the system with smaller sensitivities will be better.

At the end of this stage, the designer will have picked a single candidate for advancement to detailed design.

(c) Detailed Design

Detailed design begins with (or in a sense is preceded by) "generic optimization". The purpose of generic optimization is to yield a design that serves as a starting point for refinements (to the decision variables) to be carried out during the detailed design. One reason the generic initial optimization is needed is because the detailed design and selection of different systems, subsystems and devices will be carried out by "independent" design teams.

(i) Generic Optimization

This step is performed by the "overall-system design team"[7].

The selected candidate design is "optimized" generically. Here the optimization should include device-level variables. Ideally, this would be accomplished using software, which includes cost and performance models.

---

[7] The different teams, as referred to here, may in fact consist of only a single individual or there may be overlap in personnel. A team may even be from a different firm.

In the absence of such software, the initial, "optimized" design could be a design penned by an engineer with sufficient "fingertip-feel" to produce a good design. This "optimization" could simply be the existing design of a current, state-of-the-art vehicle, a perturbation of such an existing design or a composite of several existing designs.

In this step, exergy costs, expressed in the units of the objective function, associated with mass or power streams need to be determined. Also, each device in the design needs to have its "optimized" parameters listed. Examples of these parameters are capital costs, efficiencies and weights.

(ii) Subsystem and/or Device Design and/or Selection

The optimized parameters from generic optimization *(i)* are given to the "device design teams" as initial design values, along with the relevant exergy costs and an objective function for their device, which includes only local variables. System-level decision variables are to be considered as constraints. Device design teams are responsible for the detailed design of a device or the selection of a device. The designers are encouraged to better the results (e.g. cost, weight, efficiency of their device) from the initial system design and optimization, striving for the optimum of the local objective function. (The method for developing the local objective function will be presented shortly.) Thus, the generic design's optimized device-level decision variables serve as a point of departure for the detailed design.

Several different scenarios are foreseeable for a design. For examples:

- The device will be designed in-house.
- The device, for which a design team is responsible, is to be purchased from an outside source as an "off-the shelf" component. One example: An architect-engineer firm is designing a combined-cycle power plant. The team responsible for the gas turbine engine will be selecting an existing model from a firm, which specialized in building such engines.
- The device is to designed by an outside firm. Here the outside firm becomes the device design team, and should be supplied with the same information that an in-house design team would receive. Heat exchangers are an example of a device, which often fit in this category.

(iii) Perturbations

There may be instances where a device design team may need or propose to perturb system-level variables. Also, in the search for improvements, there may be times when it is desirable to do so. For example, it may propose to drop an operating pressure somewhat, because a drastic reduction in weight (or material cost) could be achieved, due to materials being often available only in discrete sizes. When this happens, the device team must consult with the system team. Decisions can be facilitated by rerunning the simulation or using the results of the sensitivity analysis.

(iv) Simulation and Proofing

After all initial device designs are completed the simulation is rerun with models of the proposed equipment to ensure proper on and off-design operation. At this point, it is desirable to globally optimize once again any operating variables. If these do not vary from the values resulting from the initial, generic optimization, the thermal design is, for all practical purposes, finished. It is even conceivable that further iterations of the detailed design, with updated exergy costs, would diverge further from the theoretical, optimal design. This re-optimization puts such variables in line with the actual devices used, as opposed to being optimized for generic, non-existent equipment.

If in the re-optimization of operational variables, noticeable excursions from the initial design values occur, it may be desirable to reconsider the global variables and repeat steps 2 through 4.

(v) Updating of Generic Models

Device design will be documented so as to improve and update weight and cost models. Now, methods for developing the objective functions (overall, and device) referred to above will be proposed.

V. DEVELOPING AN OVERALL OBJECTIVE FUNCTION

(a) A Proposed Overall Objective Function

Before beginning any design, an engineer (or team) must determine the constraints and objectives of the project. For a meaningful optimization the objectives need to be expressed in a single function.

A traditional objective function for an overall energy system with a single feed and single product is (El-Sayed, 1995)

$$J = c_f F + \sum Z - v_p P \tag{4}$$

where J is to be minimized ($-J$ is the profit), F symbolizes the amount of feed, Z symbolizes capital equipment and P is the product. The lower case c's and v's represent value, per unit, of the feed and product, respectively.

Although a vehicle has energy feeds, its product is performance rather than energy. Nevertheless, equation (4) remains appropriate for the overall vehicle, when

$$J = c_f F + \sum Z - \sum v_p P \tag{5}$$

Each P is now used to signify the various performance benefits. For the case of an aircraft, P's might account for climb rate and cruising speed, for examples.

Taking equation (5) and negating it yields a "profit" to be maximized, which will be referred to as Π.

$$\Pi = \sum v_p P - \left[c_f F + \sum Z\right] \tag{6}$$

This objective function is useful in conceptual and preliminary design, but cannot directly be applied to systems or subsystems, without simulation of the entire vehicle. The system or subsystem will have inputs and outputs that are mass, momentum or entropy streams. In order to evaluate such a system or subsystem, (6) must be translated into a function that contains terms for the exergy flows are being produced/consumed. This process will be demonstrated later.

(b) Determining the Value of Performance

Clearly, before one can apply (6) to a conceptual design, one must find the numerical values of the various v_p's. These values are, in the most general case, functions of the levels of performance themselves. One would expect a very low level of performance for a given desideratum would be zero, that is, it is unacceptable.

As an example of this, consider an automobile. A person residing in the United States (with typical speed limits in the 100-125 km/hr range) might be unwilling to consider purchasing a vehicle incapable of at least 125 km/hr.

However, this person would likely be willing to pay more for an automobile that could achieve a higher speed than this minimal amount. Speed limits (or more correctly, the enforcement thereof) cause a reduction in the marginal value of a 1 km/hr increase in speed as the top speed of the car becomes higher and higher. This hypothetical person, therefore, might not be willing to pay any more money for an automobile capable of reaching 240 km/hr than an automobile capable of reaching 200 km/hr. (Acceleration would be a separate desideratum.)

Returning to the example of the fighter plane, one can imagine that a certain level of overall performance could be reached that would insure victory over any opponent and penetration of any airspace. It is not rational to invest further resources in performance beyond this point. So we expect that the functions $v_p(P)$ become flat after a certain value for a desideratum is reached. In between these points (the minimum acceptable and the maximum useful) lies some continuous function.

A method is laid out below to estimate these functions. It consists of five steps, and may involve iteration. It should be noted that in order, ultimately, to optimize a *subsystem*, it is absolutely necessary to develop this information, that is, the functions $v_p(P)$, in some form, even if it is not done with the method used in this dissertation.

(i) Step 1: Determining Median Performance

The foregoing algebraic "tradeoff functions" $v_p(P)$ for representing the value of performance may have an arbitrary shape between the points of minimum acceptable and maximum desirable performance. However, if a limited range of performance is considered, the assumption of a linear relationship is reasonable. One way in which this linear function may be constructed is around a median point. If a linear function is undesirable, this information will still be of use in the construction of the function.

For a military combat aircraft, one way to find a median point is by considering the performance of the aircraft's adversaries, both current and projected[8]. For each desideratum, at least one of the aircraft in the adversarial group has a best value. That value could be selected as the median of acceptable performance levels. The set of medians would form a "standard" of comparison for further investigations.

The same basic idea is applicable to civilian vehicles. However, the median values of a market segment might instead be chosen to set the "standard values."

(ii) Step 2: Projected Units Costs and Projected Production

A realistic estimate of both the cost per vehicle and the total production quantity of the vehicle should be made. As will be seen below, this step is an aid to the fourth step.

---

[8] Here, the aircraft's adversaries are considered to be other aircraft. This view could of course be expanded to consider enemy air defense capabilities.

(iii) Step 3: Projected Research, Design and Development costs

The projected research, design and development costs should be listed. This is also an aid to the following step. When employed with the information attained above, the total project cost may be estimated.

(iv) Step 4:Algebraic Tradeoff Functions

The algebraic tradeoff functions for each desideratum must be determined through questioning of the customer or end-user. As a bare minimum, the following three questions should be asked (in some form).

1. What is the minimum acceptable value for each of the desiderata?
2. Is there a point beyond which further improvement is not necessary?
3. How much would a given improvement, over and above the median value determined in Step 1, be valued?

With the answers to these three questions determined, the simplest tradeoff function, linear, may be determined. Once again visiting the automobile example, let us imagine that the marketing department has asked the above three questions to potential customers regarding the top speed of an automobile of a certain class. The average (or weighted average) answers were, S=125 km/hr as a minimum acceptable top speed, S=200 km/hr as a ceiling beyond which improvement has little or no value, and a willingness to pay 800 dollars for an improvement ΔS=10 km/hr over a median 165 km/hr top speed. One can imagine a function, v_s(S), which would have a value of zero dollars up to 125 km/hr and rise with a slope of 80 dollars/(km/hr) to a maximum of 6000 dollars.

However, for many cases the questions may not be best asked in terms of dollars. For a military vehicle, say an air superiority fighter, dollars would be a poor choice of units. In this example, the end-user (the Department of the Air Force or Navy) is not the same as the purchaser (Congress). The purchase costs have reached such high amounts that it is difficult for the average person to comprehend the sums in rational terms. Furthermore, neither body (Congress or the end-user) is spending their own money.

In such a case the questions may be rephrased in terms of production sacrifices; Question Number 3 could be changed to: "What reduction, in number of aircraft delivered to you, would you accept in order to obtain a specified improvement from the median value determined in Step 1?" A military leader should have a good grasp of tradeoffs between quantity and quality. The information from Steps 2 and 3 allows production tradeoffs to be converted to a dollar amount (or a cost to a production adjustment).

If a computer simulation were to be available that would predict aircraft survivability as a function of measured desiderata, it could be used to develop, or help develop, these trade-off functions.

(v) Step 5: Relative Weighting

At the overall vehicle level, the algebraic tradeoff functions are sufficient for optimization. As shown below, in order to decompose the vehicle into systems and subsystems, weighting factors will be needed. Then a distinct objective function can be defined for each system, subsystem or device, each to be designed by a distinct "team".

The relative weighting factor for the desideratum P_i, as employed in this dissertation, is deduced from the foregoing information and is given by

$$W_i = \frac{\dfrac{\Delta\$_i / \$}{\Delta P_i / P_i}}{\sum_k \dfrac{\Delta\$_k / \$}{\Delta P_k / P_k}} \tag{7}$$

Where $ represents total money, P a performance desideratum, and ΔS the increase in price an end user would pay for a ΔP increase in a desideratum. The numerator, then, represents the percent of expenditure increase the customer is willing to make, per percent increase in a given performance desideratum P_i. The denominator is the sum over all desiderata; so W_i represents the relative importance of P_i. S cancels, leaving

$$W_i = \frac{\dfrac{\Delta\$_i}{\Delta P_i / P_i}}{\sum_k \dfrac{\Delta\$_k}{\Delta P_k / P_k}} \tag{8}$$

(vi) Iteration

It may be necessary to repeat Steps 1 through 5 as a preliminary design is completed. This is due to an implicit assumption as to the independence of the individual desiderata. An end-user may sacrifice far more total resources than intended, as this person was looking at only a single desideratum at a time when proceeding through the five steps. Or, the standard performance may be so great that the user erred in the opposite direction. Therefore, there must be good communication between designer and end-user at all times. As a conceptual design evolves, the questions from Step 3 may need to be repeated. A design may have become, in the end-users opinion, too expensive; or its performance may simply be inadequate. This is especially true if the median performance was far from the optimum.

(vii) Notes on the Development of an Overall Objective Function

In the methods presented here, the units of each term in the objective functions are monetary. For commercial vehicles and transport vehicles, monetary units are the obvious choice. For combat vehicles, other units may be better, such as "production quantity".

VI. THERMOECONOMICS AND DETAILED DESIGN

(a) Concurrent Engineering and the Need for Decomposition

Once a conceptual design has been accepted, the detailed design should proceed in an efficient fashion. It is not possible to optimize and design an aircraft, or even an energy subsystem, as an entirety. Vehicular energy systems are too complex to be designed by a single individual or team, as the number of decision variables becomes unmanageable. Therefore the design or selection of individual devices and/or subsystems is delegated to subordinate teams or individuals. *It is desirable for these designers to have a methodology and information that allows them to make decisions in accordance with the overall goals of the vehicle.* Furthermore, it must be determined which decision variables are local and which are global. That is, which variables a device design team is free to vary in optimizing its individual device, and which are constraints to the design team so as not to affect the designs being carried out, simultaneously, by other design teams. These determinations may at times be made through "common sense", but at other times may require a sensitivity analysis.

A vehicle typically relies on one fuel to achieve its goals. Because the subsystem or device design team is not optimizing a whole vehicle, but something which may produce or consume commodities not considered when looking at the whole vehicle, its objective function will vary from that of the overall vehicle. For example, the alternator on a light general aviation aircraft does not *directly* consume fuel, nor does it directly influence performance. Nonetheless, an aircraft with an alternator that is both lighter and more efficient, will, with all else remaining equal, perform better (and cost more). The person selecting an alternator should not be burdened with, nor at this stage of the design be necessarily capable of, directly calculating the impact upon aircraft performance

The alternator mentioned above still has only one product (electrical energy). An onboard energy device or subsystem may use several "fuels" and/or supply several "products". These fuels and products, besides electricity and shaft power, include, but are not limited to, compressed air, hydraulic power and heat (or cooling). Additionally, one must account for the lift required to hold the device in the air (or to make it go up). The costs and benefits of these mass and energy flows must be included in the device or subsystem design team's objective function.

Before proceeding in developing an objective function, for a device or subsystem, which is to be designed by a team subordinate to the overall-vehicle design team, methods are needed for determining the costs and benefits. This is done through accounting for mass and energy streams with exergy, and using the methods of thermoeconomics to place unit costs on them.[9]

(b) The Role for Exergy and Thermoeconomics

The role for exergy and thermoeconomics discussed within this paper is for employment during detailed design. In this paper detailed design is the "filling in" of black boxes such as a propulsion system, environmental control system or a hydraulic pump. The methodology presented here assumes that the overall-vehicle designers have penned a "reasonably optimized" preliminary design. Additionally, it is assumed that allowance has been made for the weight and exergy consumption of systems and subsystems. Before employing this thermoeconomic methodology, it is absolutely necessary to have the previously defined overall objective function, weighting factors and values of performance. Thus all design teams will strive for the same common goals.

(i) Exergy

For comparing energy converters that have single feed and product streams, and perform the same function, performance measures such as "thermal efficiency" and "COP" often suffice. Such energy-based performance measurements are referred to as first law performance measures. As devices or systems add additional feeds and/or products, these definitions become inadequate. Moreover, even if a device has a single feed and a single product, but is one component in a complex system, such first law measures are inadequate for comparing the relative impact of any one device upon the overall system performance. That is, first law measures of feeds and products do not represent the true values of these commodities. What is needed in analyzing and optimizing complex systems is a common, consistent measure for representing the potential of mass and energy flows.

This proper method of quantifying fuels and products is with exergy (sometimes called "availability" or "available energy"). Exergy measures the ability of a commodity (matter, momentum, entropy or "heat", charge...) to cause change.

[9] As a general reference to exergy and thermoeconomics see, for example, Bejan et al., 1996.

After preliminary design, all exergy flows and destructions within the proposed vehicle should be diagrammed. As a vehicle may be evaluated under several different operating modes (typically related to the desiderata), it may be necessary to create several exergy flow diagrams. If one were developing a fighter aircraft, diagrams may be necessary for cruise, maneuver, climb, etc.

Exergy is dependent on the environment in which the vehicle operates. If the range of operating conditions is not too large, an average (or weighted average) of the operating conditions may suffice to serve as the reference environment. For vehicles operating in a wide variety of conditions (i.e., aircraft) a number of diagrams, sufficient to cover the variations in operating environment, will be necessary. It may even be necessary to express values as functions of time for such realms of operation as a maximum performance climb, where ambient pressure and temperature are changing rapidly.

The purpose of these exergy diagrams is twofold.[10] The first benefit is that this diagram allows the designers to truly comprehend the interactions and inefficiencies of an energy system. The relative magnitude of the destructions (and wastes) will point to the areas of a system where improvements will be most beneficial. This topic will not be addressed directly here, but is addressed extensively in the literature. The second benefit of creating the exergy flow and destruction diagram is that costs may be assigned to each stream's exergy flow. That is, the cost per unit of exergy can be evaluated at each station on the exergy flow diagram. Thus, not only exergy, but also costs can be tracked through the overall vehicle. Moreover, the unit costs of exergy entering a subsystem (or device) are of special relevance to that subsystem's design team. Before proceeding to the construction of an exergy flow diagram for an aircraft, the methods for establishing the unit costs, thermoeconomics, will now be discussed.

(ii) Thermoeconomics

Thermoeconomics allows a cost to be associated with each exergy flow. These unit costs are absolutely necessary in the development of the objective functions for an on-board device or subsystem.

After the overall exergy flow diagrams are finished, complete exergy costing and cost flow diagrams should be created for each exergy flow diagram. Capital costs should not be included in the costing; as the local optimization should be changing design values only by relatively small amounts, the proper monetary charges are for marginal costs. (This is explained in more detail in Section 3.06.)

For a subsystem, the costing will be performed one of two ways, depending on whether it is at a maximum performance condition or a "cruise" condition. These correspond to costing for fixed resources or fixed output, respectively (El-Sayed, 1995). In a maximum performance condition, the vehicle's primary energy conversion system (i.e., its propulsion system) is producing as much power as possible. Any exergy drawn to fuel the subsystem will be detracting from the vehicle's performance. Therefore all exergy flows must be assigned a per unit cost based on the cost to vehicle performance. For example, a light aircraft in full throttle cruise at a specified altitude, where the desired performance is a cruise speed, can have a marginal value of thrust exergy assigned with

$$\nu_T = \frac{\partial V}{\partial \dot{X}_T}\frac{\partial \Pi}{\partial V} \tag{9}$$

[10] See, for example Moran, 1989 and Szargut et al., 1988.

Here, $\frac{\partial V}{\partial \dot{X}_T}$ is the partial derivative of velocity with respect to thrust exergy and $\frac{\partial \Pi}{\partial V}$ is the slope of the cruise speed "tradeoff" curve. Multiplying these two partial derivatives yields $\frac{\partial \Pi}{\partial \dot{X}_T}$, which clearly is the marginal profit associated with thrust exergy.

This information allows the complete exergy costing to be completed.

On the other hand, for the case of "part-throttle" cruise, costing is based on the cost of fuel Engine power is no longer the limiting commodity; any drop in performance may be accommodated by increasing the power output of the engine. One resulting cost is the direct cost for additional fuel. Furthermore, if range is a desideratum, the cost of an increase of fuel burn to range may be added to the fuel's base cost. This will in general be

$$c_{f,range} = -\frac{\partial R}{\partial \dot{m}_{fuel}} \frac{\partial \Pi}{\partial R} \tag{10}$$

If a device is a member of the vehicle's propulsion system (e.g., a piston engine in a propeller driven aircraft), it will be shown that exergy costing will have to be done from both of the preceding viewpoints.

The application of the second law of thermodynamics, combined with economics, allows for decomposition of systems into devices and subsystems that may be independently optimized. This theory has been developed and validated by El-Sayed and coworkers. (See for example, El-Sayed and Evans, 1970; El-Sayed, 1995.)

(c) The Objective Function for an on-board Subsystem

With the exergy flow and cost diagrams complete, there is enough information to optimize a subsystem. Here a subsystem is defined as an on-board energy conversion system *not directly involved in producing the necessary basic forces of flight* – thrust and lift. The subsystem is, in general, required to deliver a certain output as a design constraint. Then, the local objective function for a subsystem, to be *minimized*, is a total cost.

$$J = Z + \sum_i W_i \left(c_m m + \sum_{i,j} c_{i,j} \dot{X}_{i,j} \right) \tag{11}$$

Here, i represents each desideratum, j each feed stream, $c_{i,j}$ are unit costs of exergy, c_m is the unit cost of mass, and m is the mass of the component. For an aircraft, the cost of mass is related to the exergy of lift. As was stated previously, this exergy expenditure used to hold mass in the air must be taken into account. Suitable relations will be given in the example that follows. W represents the various weighting factors, determined according to the procedure described in Section 3.04.

In general, costs will be functions of the decision variables themselves. For example, the marginal cost of weight on-board an aircraft is a function of the aircraft's gross weight. Therefore, as a subsystem designer strays from the estimates made about the subsystem during preliminary design, iterations may be necessary, so as to update exergy and weight costs.

(d) The Objective Function for a Propulsion System Component

While a subsystem, in general, will have its required output a design constraint, the propulsion system itself is a balance between performance supplied to the aircraft (as thrust),

weight, capital costs, range and operating costs. Its objective function differs from that of a subsystem in that there is now a profit to be maximized – the difference between the performance delivered and the cost of providing it. This takes the form of a "profit" to be maximized:

$$\Pi = \sum_i W_i \left(\sum_{i,j} v_{i,j} \dot{X}_{i,j} - c_{mi} m \right) - Z - \sum_i T_i \sum_{i,k} c_{i,k} \dot{X}_{i,k} \tag{12}$$

It is necessary to cost exergy in both manners. That is to say, costing will be performed once as if the input (fuel) were fixed, and once as if the output (performance) were fixed. Exergy outputs, with subscript j, are determined by placing a value on thrust. Inputs, with subscript k, are determined using the price of fuel (including, if necessary, cost adjustment for range). The variable T is a time factor, the percentage of time a vehicle spends in each operating mode. This follows from the overall vehicle's objective function; we charge a device in the propulsion system for only the fuel consumption for which it is responsible. In contrast, performance is available all of the time; therefore the output values are multiplied by the weighting factors of the various desideratum.

VII. NOTES ON EXERGY FOR THE DESIGN PROCESS

(a) Exergy of lift and exergy flow in aircraft

(i) Exergy of Lift

Although the exergy produced by the wing in *level* cruise (at the expense of drag, i.e., through an input of exergy with momentum) is ultimately destroyed, it is necessary to calculate the value of the lift exergy so as to properly charge individual components for their lift requirements.

In general, for any force, the exergy transfer associated with it may be defined as

$$\dot{X}_{force} = \mathbf{F}(\mathbf{V} - \mathbf{V}_0) \tag{13}$$

During steady state flight, the force of lift is equal to the weight of the airplane (or component being lifted). The problem remaining is to properly define $\mathbf{V_0}$. An ideal wing, producing no entropy, travelling forward at speed V_x will fall towards the earth at a certain rate, V_{y0}. This is a thermostatic state (Gaggioli et. al., 1999), and will be used as a dead state here.

McCormick (1995) states that the minimum induced drag coefficient for a wing is

$$C_{D,i,\min} = {C_L}/{\pi a} \tag{14}$$

where a is the aspect ratio. Therefore the minimum thrust required to generate lift is given by

$$\mathbf{F_D} = \frac{\rho A \mathbf{V}^2 C_L}{2\pi a} \tag{15}$$

The power to maintain level flight with this ideal wing is then, in accordance with equation (15)

$$\frac{\rho A V_x^3 C_L}{2\pi a} = P_{req,ideal} \tag{16}$$

The rate of climb for a typical light aircraft is well approximated with

$$V_y = \frac{P - P_{req}}{mg} \tag{17}$$

The use of (13), (16) and (17) yields the following expression for the exergy of lift.[11]

$$\dot{X}_{lift} = mgV_y + \frac{2(mg)^2}{\rho V_x A\pi a} \tag{18}$$

The marginal cost of mass is then

$$c_m = \frac{\partial \dot{X}_{lift}}{\partial m} c_{lift} \tag{19}$$

Note that c_{lift} is not the coefficient of lift, but the unit exergy cost of lift, determined via exergy costing (thermoeconomics). Performing the differentiation yields

$$c_m = \left[gV_y + \frac{4g^2 m}{\rho \mathbf{V} A\pi a} \right] c_{lift} \tag{20}$$

(ii) The Creation of an Exergy Flow Diagram

The expressions for the exergy of lift allow exergy flow diagrams to be developed. In creating these diagrams, the following points should be remembered:

- The general source of exergy is the fuel. In general, this will be converted in the propulsion system to thrust, shaft power, compressed air, etc.
- Thrust is used to produce lift (overcome induced drag), overcome parasitic drag, overcome trim drag[12] and provide ram air. In the exergy flow diagram, thrust exergy will flow out of the aircraft and back into various portions of the aircraft.
- Lift flows from the wing to the remainder of the aircraft. The amount of lift an item receives is assumed proportional to its weight. As this is definitely the case for climb, it seems reasonable to assume this for all conditions. If the aircraft is at a steady altitude, the exergy of lift is consumed by the "aircraft pieces" in the process of holding them aloft.
- These exergy diagrams will vary with the loading of the aircraft. Therefore, the exergy flow diagram will continually change during flight. An exergy flow diagram for the average flight condition, although a good start, may not always be adequate.

Simple exergy flow diagrams for the Glastar aircraft are shown in the Case Studies.

---

[11] An exergy balance on a mass in flight takes the form $dX/dt = mgV_y = \dot{X}_L - \dot{X}_\delta$. Substitution of (13) yields $\dot{X}_\delta = -mgV_{y0}$.

[12] In order to control the airplane and overcome the moment produced by the wing, fixed-wing aircraft generally have a stabilizer. Drag produced in producing the lift of this stabilizer (which actually normally is directed *downward*) is called trim drag.

(b) Choosing the "Reference Environment" for Design and Costing

For exergy calculations, it is necessary to choose a reference environment. For a stationary plant, if the conditions of its surroundings change significantly during its operation, it may be necessary to integrate over a variety of environmental conditions, or to weight, by percentage of time in a given environment, finite blocks of conditions using average conditions for that block (say, with the temperature from 0-5°C, 5-10°C and so on). An aircraft, as stated previously, operates in a wide variety of conditions. Not only will different reference conditions be used for different modes of flight (cruise, climb, etc.), but also it may be necessary to integrate over changing conditions in a single mode of flight. An example of this is a climb to cruise altitude.

However, it will be argued here that the outdoor environmental conditions are not always correct as a reference "dead state" for evaluating a system or accounting. This argument will be made through the example of a candidate system for Scenario 2 in 3.03, the "Beer and Soup" plant. Figure 3 shows this system operating at its design condition[13], along with associated exergy flows and cost flows, with a reference temperature of 288K. The management of the "Beer and Soup" plant evaluated the different systems available to them, and decided that this particular system was in the plant's best interest.

The "Beer and Soup" company's cost accountants require that both the beer department and the soup department pay for their exergy. However, it is observed that as the outdoor temperature changes (which is being used as a reference temperature) exergy charges will vary to the departments, as shown in Figure 4[14]. The department heads argue that the charges should be the same regardless of the weather. They say that their beer and soup are always essentially the same temperature, and the system operating cost is not varying. Furthermore, the management of the "Beer and Soup" company evaluated the different systems available to them, and decided that this particular system was in the plant's best interest. The system had been chosen as the best overall for the company *as a whole*.

---

[13] The thermodynamic and thermoeconomic models are given in Appendix B. Note that the turbine efficiency is zero. The author argues that the proper method for costing when a valve is used in lieu of a turbine is as a turbine with zero efficiency. This method allows capital to be played off against efficiency during the generic optimization.

[14] The thermodynamic model and the costing equations are given in Appendix B.

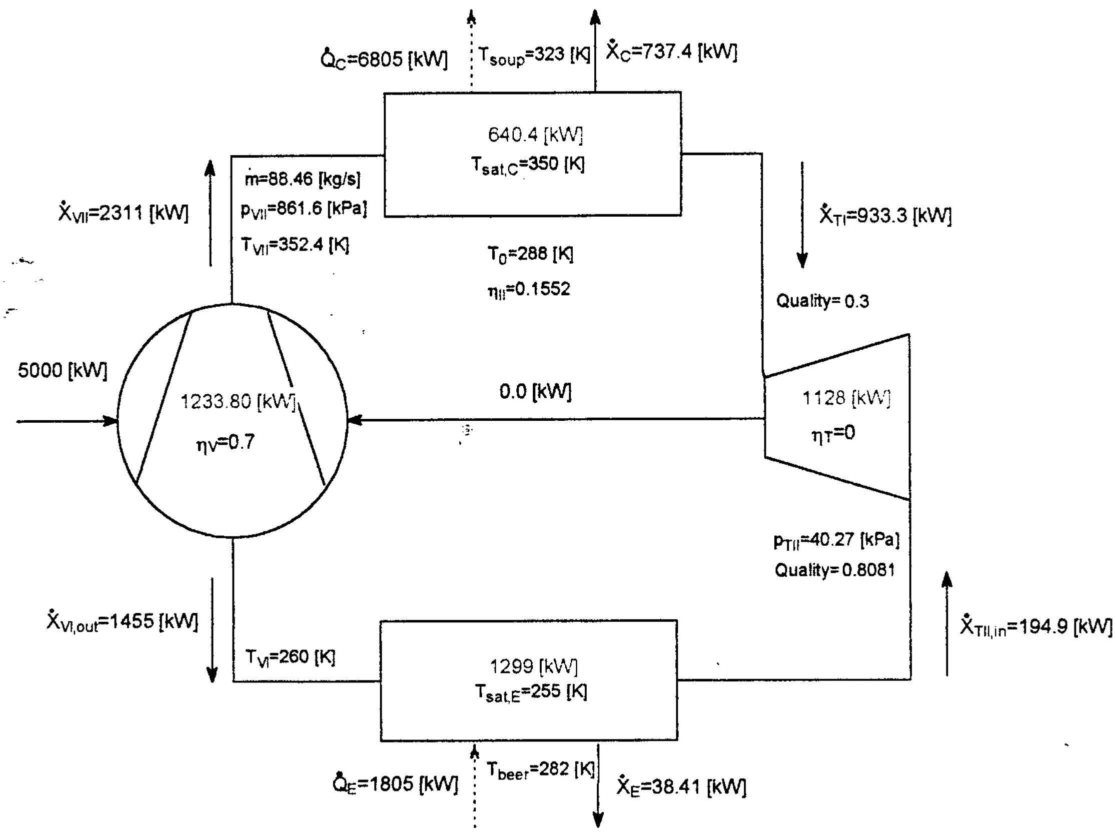


Figure 3: "Beer and Soup" Design-Point Exergy Flows

The available energy theory of Gibbs (Gaggioli et. al., 1999), along with an examination of the system's constraints, resolves the apparent paradox of varying charges to the departments without a variance in the overall operating cost of the system.

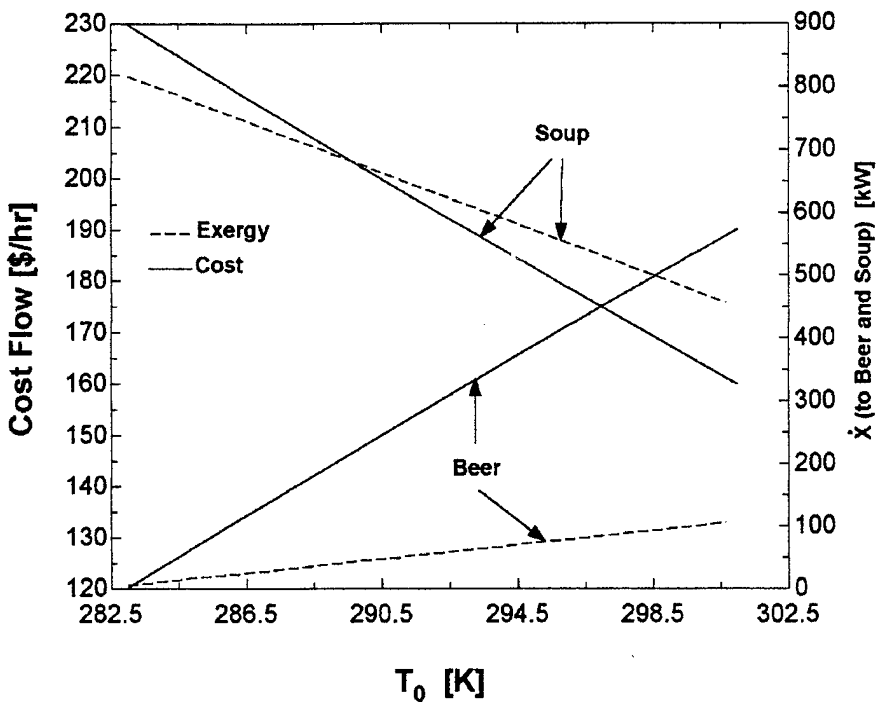


Figure 4: Variation in Exergy and Cost Flows with Dead State Temperature

(i) System Information

Before moving on, it will be necessary to have more information about the beer and soup plant. At base load, it is designed to cool 31.9 kg/s of beer from 288K to 276K and heat 33.3 kg/s of soup from 300K to 347K. (Calculations of the log mean temperature of each will yield the temperatures of the beer and soup from previous examples of this plant.) The plant's circulating fluid, R114, has a mass flow of 88.5 kg/s and operates between 862 and 40.3 kPa. The compressor requires 5000 kW of shaft power and has an isentropic efficiency of 70%.

(ii) Employing Gibb's Available Energy Theory

At any instant, available energy for an object, or collection of objects, is defined by Gibbs as

$$A = E - E_{min}\left(Constraints\right) \tag{21}$$

Given the instantaneous values of the constraints, the available energy is obtained by reducing the energy to the minimum possible values consistent with the constraints. The "dead state" is the minimum-energy state, the constrained equilibrium state, at these values of the constraints. Therefore, in order to find the dead state, the constraints must be defined.

1) Constraints

The beer and soup plant never interacts thermally with the ambient environment. It interacts with only the beer and soup department. Its performance (i.e., power requirement and exergy delivery) depends only on the beer and soup temperatures. The composite of the beer and soup entering the system has available energy because the beer and soup are not in equilibrium

with each other. The purpose of the plant is to increase the temperature of the soup and decrease the temperature of the beer, driving them further out of equilibrium and hence increasing the available energy of the composite.

A typical refrigeration system's working fluid will be in a two-phase state at normal operating temperatures when the system is not running. Therefore, the dead state of the refrigerant should be saturated at the reference temperature. (Wepfer and Gaggioli, 1980)

Finally, chemical exergy should not be considered for this system. As long as the refrigerant is contained inside of the system, it is constrained from chemical reactions.

2) Calculation of the Equilibrium State

If we assume the two fluids are incompressible and have constant specific heats and assuming a reference temperature of absolute zero, the energy flow may be defined as

$$\dot{E} = C_{beer}T_{beer} + C_{soup}T_{soup} \tag{22}$$

where $C = \dot{m}c_p$. Then $\dot{E}_{min}$ is found by minimizing (22) with the constraint of the entropy flow remaining constant. The constraint yields

$$C_{beer}\ln\frac{T_{beer}}{T_{beer,eq}} + C_{soup}\ln\frac{T_{soup}}{T_{soup,eq}} = 0 \tag{23}$$

Solving equation (23) for $T_{soup,eq}$ and substituting into (22),

$$\dot{E} = C_{beer}T_{beer,eq} + C_{soup}T_{soup}\left(\frac{T_{beer}}{T_{beer,eq}}\right)^{\frac{C_{beer}}{C_{soup}}} \tag{24}$$

The minimum energy is obtained by taking the derivative of this with respect to $T_{beer,eq}$ and setting it equal to zero.

$$0 = \frac{\partial\dot{E}}{\partial T_{beer,eq}} = C_{beer} - T_{soup}\left(\frac{T_{beer}}{T_{beer,eq}}\right)^{\frac{C_{beer}}{C_{soup}}}\frac{C_{beer}}{T_{beer,eq}} \tag{25}$$

Solving for $T_{beer,eq}$ yields

$$T_{beer,eq} = T_{soup}\left(\frac{T_{soup}}{T_{beer}}\right)^{-\frac{C_{beer}}{C_{beer}+C_{soup}}} \tag{26}$$

As expected, when this result is substituted into (23) to get $T_{soup,eq}$ the same result is obtained. For the conditions of the beer and soup entering the plant, this temperature is 294K. For the beer and soup exiting the plant it is 310K. In practice, either of these would suffice as an appropriate dead state temperature; in theory a proper average is required.

3) Calculation of the Theoretical Dead State Temperature

The reason that the equilibrium temperature changes, as the beer is cooled and the soup is heated, is because the increase in available energy of the beer-soup composite is less than the net energy input (the compressor power). That is, because $\dot{E}_{\min}$ is increasing as a result of entropy production. The change in $\dot{E}_{\min}$ is given by $d\dot{E}_{\min} = T_{eq}d\dot{S}$ where $\dot{S}_{out} - \dot{S}_{in} = \left(\dot{S}_{soup,out} - \dot{S}_{soup,in}\right) + \left(\dot{S}_{beer,out} - \dot{S}_{beer,in}\right)$ equals the overall rate of entropy production. Thus,

$$\Delta E_{min} = \dot{E}_{min,out} - \dot{E}_{min,in} = \int_{\dot{S}_{in}}^{\dot{S}_{out}} T_{eq} dS \tag{27}$$

Hence the appropriate average equilibrium temperature is equal to

$$T_0 = \Delta\dot{E}_{min} \Big/ \Delta\dot{S}_{min} \tag{28}$$

This temperature is that of a hypothetical entropy reservoir from which the entropy $\dot{S}_{out} - \dot{S}_{in}$ could be obtained (i) while transferring a net amount of energy $\dot{E}_{min,out} - \dot{E}_{min,in}$ to the system (ii) *with no net transfer of available energy* used in order to change $\dot{E}_{min}$. This is illustrated in Figure 5; the area under the curve is $\Delta\dot{E}_{min}$. When T_0 equals $\Delta\dot{E}_{min} \Big/ \Delta\dot{S}_{min}$, the two shaded areas are equal. Available energy obtained while raising T_{eq} from $T_{eq,in}$ to T_0 is returned upon raising T_{eq} from T_0 to $T_{eq,out}$.

Finally, inasmuch as

$$\dot{E}_{min,out} - \dot{E}_{min,in} = C_{total}\left(T_{eq,out} - T_{eq,in}\right) \tag{29}$$

and

$$\dot{S}_{min,out} - \dot{S}_{min,in} = C_{total} \ln\left(T_{eq,out} \Big/ T_{eq,in} \right) \tag{30}$$

$$T_0 = \frac{C_{total}\left[T_{eq,II} - T_{eq,I}\right]}{C_{total} \ln T_{eq,II} \Big/ T_{eq,I}} \tag{31}$$

This is the logarithmic mean of the equilibrium temperatures, as calculated by (26), before and after the process occurs. For this example, that temperature is 302K. The corresponding exergy flow and costing is shown in Figure 6.

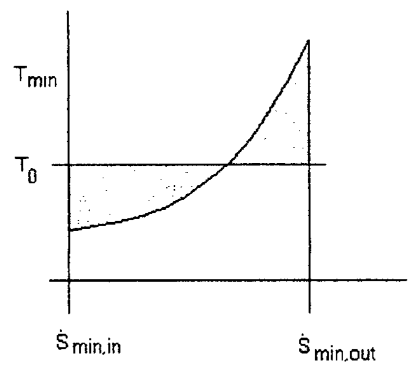


Figure 5: Theoretical T-S Curve

As a closure to the "beer and soup" story the following should be mentioned. After the previous calculations had been performed, the beer department manager, well-trained in exergy, calculated that his heat exchanger efficiency was only 5.5% while his colleague's, in the soup department, had an efficiency of 26%. He asked, "Why should I be penalized with higher cost per unit exergy just because management stuck me with a low efficiency heat exchanger?"

Indeed, he was correct. Costing was revised to the equivalence method, with the unit price of exergy the same for both the beer and soup departments.

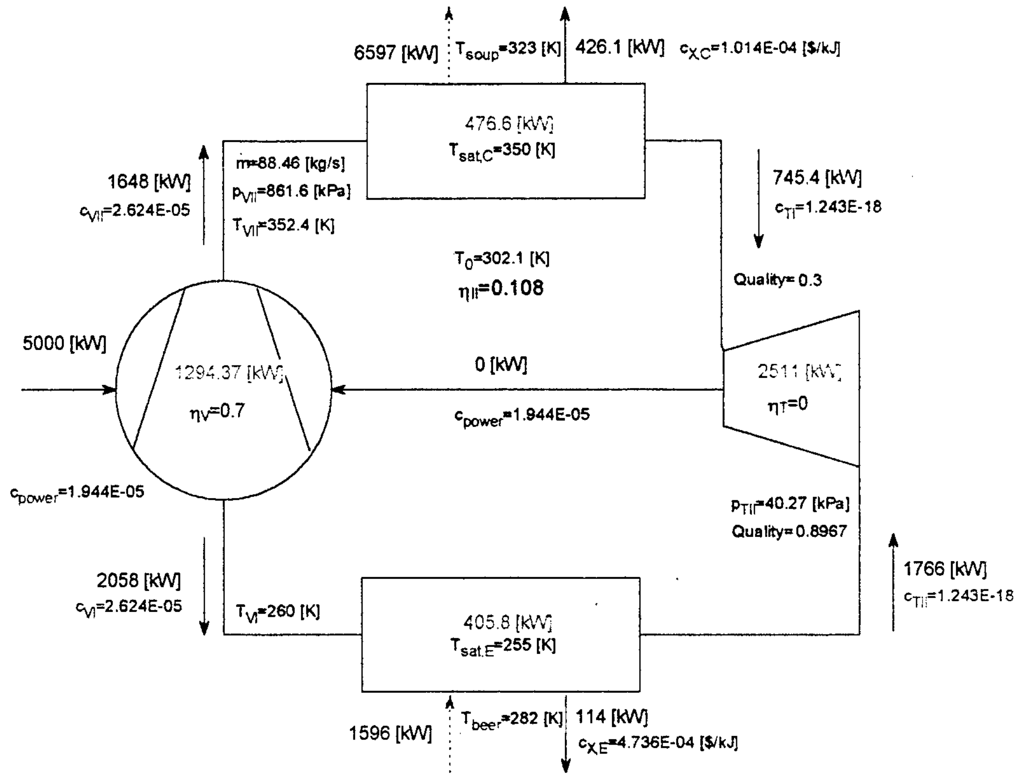


Figure 6: Exergy Flows and Unit Costs, Calculated with Equilibrium Temperature

(c) Marginal Exergy Costs

The most common exergy cost is an average cost, which includes capital costs. This cost is at times misused, such as when operating decisions are being made for an existing plan (El-Sayed and Gaggioli, 1989). At such time the proper cost is the marginal exergy cost,

$$c_i' = \frac{\partial\left(c_i \dot{X}_i\right)}{\partial \dot{X}_i} \tag{32}$$

There is another instance, essentially unnoticed before, when marginal costs should be used. The marginal cost is also the proper costing for detailed design, as during detailed design the device or subsystem design team is operating under the assumption that the remainder equipment for the system is fixed. *The team's design has no effect on the capital costs of the remainder of the plant.* These capital costs must therefore be considered sunk.

Consider once design scenario one, the aircraft example. Including capital costs in the cost equations (which will be described in detail in the case studies) yields a cost of weight of hundreds of dollars. This high of a cost is sensible during preliminary design, as reduced weight can reduce engine power requirements (as well as structural requirements). However, during detailed design, an engine, or at least range of engines has been selected. Weight reduction affect only performance. In the case studies it will be seen that not including capital cost reduces the cost of weigh by an order of magnitude. One would be hard pressed to find an owner of a light

plane willing to spend hundreds of dollars to reduce the empty weight of his plane by a couple of pounds, but many would be willing to spend tens of dollars.

The most accurate analysis would integrate marginal cost from the initial design parameters (passed to a team at the beginning of detailed design) to the actual operating point, as the marginal cost is a function of the design parameters. If deviations are small, the marginal costs may be assumed constant.

Furthermore, if the deviations from the initial design parameters remain small, incremental costs may *often* be used in place of the marginal costs. *Here*, the incremental cost is defined as the average exergy cost ignoring capital costs. This is simply the marginal cost if equipment performance is considered constant.

These points are illustrated with the following example. Consider an existing combustion gas turbine power plant. The owner requires an additional power output from the plant; it is decided to add a Rankine bottoming cycle as illustrated in Figure 7. The generically "optimized" initial design calls for "duct firing", which is combustion in the turbine exhaust before it enters the heat recovery steam generator (HRSG).

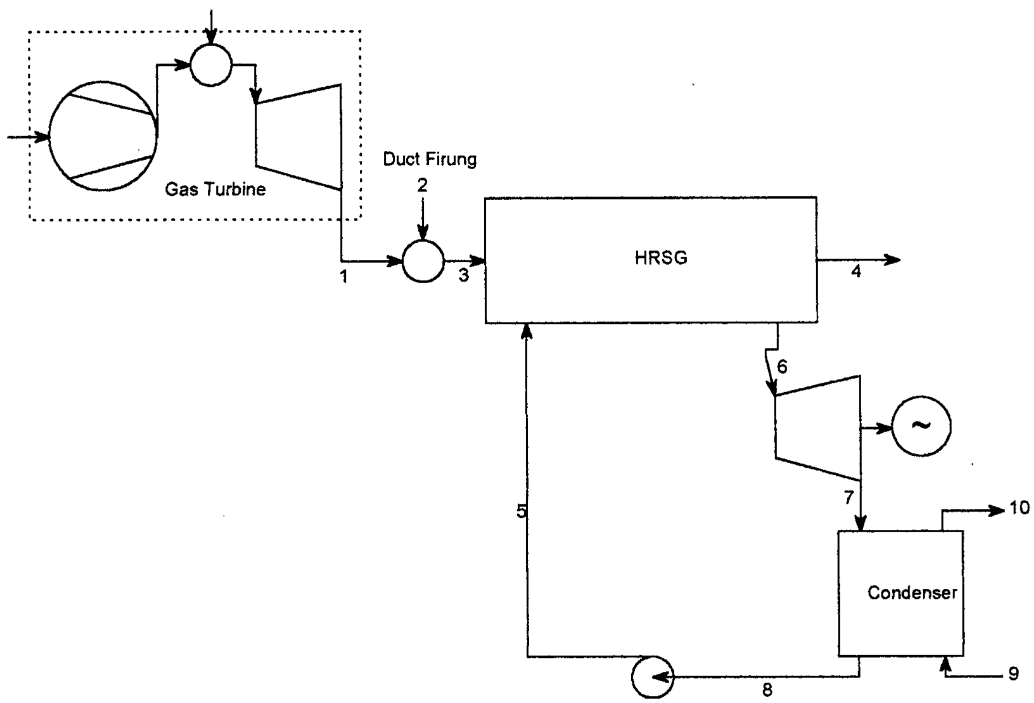


Figure 7: Rankine Bottoming Cycle

A *component* design team is responsible for selecting a suitable steam turbine-generator. They are given, from the *system* design team, the design values from the generically optimized system, such as mass flow, condenser vacuum, superheat, expected capital and exergy costs, etc. It is highly unlikely that a steam turbine exists that will perfectly match these values, nor does it take a detailed economic analysis to realize that designing and custom building a steam turbine makes no economic sense. The required output of the turbine-generator is fixed. The initial total cost, from the "generic optimization", based on the thermodynamic model and generic cost models, is

$$J_{initial} = \dot{Z}_{st,initial} + c_6 \dot{X}_{6,initial} + c_7 \dot{X}_{7,initial} \tag{33}$$

The problem of selecting a steam turbine-generator becomes finding the match that satisfies system-level constraints while, hopefully, decreasing the total cost over the initial design value, or, if that proves unfeasible, minimizing the increase in cost. With the approximation of constant marginal costs, this is written as the following function to be minimized.

$$J_{st} = \left(\dot{Z}_{st} - \dot{Z}_{st,initial}\right) + c_6'\left(\dot{X}_6 - \dot{X}_{6,initial}\right) + c_7'\left(\dot{X}_7 - \dot{X}_{7,initial}\right) \tag{34}$$

The initial design terms may be eliminated from the problem, as they have been determined in the generic optimization process.

To calculate the marginal cost of steam supplied, the cost of the steam is first calculated with a money balance around the HRSG and duct firing.

$$c_6 \dot{X}_6 = c_1 \dot{X}_1 + c_2 \dot{X}_2 + c_5 \cancel{\dot{X}_5} + \dot{Z}_{DF} + \dot{Z}_{HRSG} \tag{35}$$

It is assumed here that the feedwater exergy is negligible compared to that of the combustion gases.

The marginal cost of the steam supplied is

$$\frac{\partial\left(c_6 \dot{X}_6\right)}{\partial \dot{X}_6} = c_1 \cancel{\frac{\partial \dot{X}_1}{\partial \dot{X}_6}} + c_2 \frac{\partial \dot{X}_2}{\partial \dot{X}_6} + \cancel{\frac{\partial \dot{Z}_{DF}}{\partial \dot{X}_6}} + \cancel{\frac{\partial \dot{Z}_{HRSG}}{\partial \dot{X}_6}} \tag{36}$$

The three terms cancel because the flow of gas turbine exhaust is fixed and, at this stage, the selection of the steam turbine is being done independently from that of the HRSG and duct firing design.[15] (Changes in the amount of duct firing and flow of cooling water through the condenser will be used to compensate for deviation from the initial design.)

For this problem, the second-law efficiencies of the duct firing and HRSG are defined as

$$\eta_{HRSG} = \frac{\dot{X}_6 - \dot{X}_5}{\dot{X}_3} \tag{37}$$

$$\eta_{DF} = \frac{\dot{X}_3}{\dot{X}_1 + \dot{X}_2} \tag{38}$$

From these two efficiencies, and once again with the assumption that the feedwater exergy is negligible,

$$\dot{X}_6 = \eta_{HRSG}\eta_{DF}\left(\dot{X}_1 + \dot{X}_2\right) \tag{39}$$

Rearranging and taking the partial derivative, assuming constant efficiencies, yields

$$\frac{\partial \dot{X}_2}{\partial \dot{X}_6} = \frac{1}{\eta_{HRSG}\eta_{DF}} \tag{40}$$

---

[15] As discussed in Section 3.03, before detailed design is complete, simulation and proofing are done to ensure that independently designed or selected devices function properly together. Moreover, an additional optimization on operating parameters may be performed.

and

$$c_6' = c_2 \frac{1}{\eta_{HRSG}\eta_{DF}} \tag{41}$$

It will now be demonstrated that this is a case that *cannot* be well approximated by the incremental cost. As gas turbine exhaust is currently being "thrown away", the cost associated with it is zero. The cost balance (35) without capital cost is then

$$c_6\dot{X}_6 = \cancel{c_1}\dot{X}_1 + c_2\dot{X}_2 + \cancel{c_5}\dot{X}_5 \tag{42}$$

Substituting (41) from above, and solving for c_6 yields

$$c_6 = \frac{c_2}{\eta_{HRSG}\eta_{DF}} \frac{\dot{X}_2}{\dot{X}_1 + \dot{X}_2} \tag{43}$$

The reason for this inaccuracy is that in this case, all additional exergy as steam can only be had at the expense of additional duct firing. Equation (43) would assume that it would be split from turbine exhaust and duct firing. However, there is no physical way to increase the gas turbine exhaust[16].

Similar methodology would be used to find c_7'. After selection of the individual devices in this system is complete, the design team moves on to the simulation and proofing phase. At this stage, it would be desirable to re-optimize on operating variables. For this system, these could include superheat, operating pressure and condenser vacuum.

Marginal exergy costs are the proper exergy costs to use in detailed design. Additionally, if the last stage of a global-local optimization scheme were a local optimization, the use of marginal costs here could likely improve accuracy. In many cases, but not all, incremental costs may be used in place of marginal costs. However, if the engineer does not have a good physical sense of how the system performs, it is wise to perform some mathematical investigation as was performed above.

VIII. CASE STUDIES FOR A LIGHT AIRCRAFT

The case studies that follow are all based on scenario 1, as discussed in Section 3.03. The aircraft is the Glastar aircraft (see Appendix A for details). Here, an overall objective function will be formulated. Exergy flow and cost diagrams, as required to begin the detailed design process, are developed.[17] Then, the applicability of the methods previously developed here to the detailed design process will be demonstrated through alternator and engine selection.

(a) A User Questionnaire

In order to construct objective functions it was necessary to question the user about the value of different desiderata.

---

[16] An analagous scenario would occur when an aircraft engine must use its afterburner in order to increase thrust.

[17] When detailed design is discussed, it is assumed that "generic" optimization has selected the 160 horsepower Lycoming O-320 engine and a constant-speed propeller.

It was decided, based on the light aircraft's typical "mission" profile, to consider full power cruise at 8000 ft., economy cruise at 8000 ft. and climb rate as desiderata. Climb rate was considered a measure of short field performance. Although it would have been advantageous to add a more direct measure of this part of the mission, it would have involved using a far more complicated aircraft simulation. A summary of conditions for the measurement of the various desiderata and their preliminary design values is given in Table 6[18].

	Cargo Mass (kg)	Fuel Mass[19] (kg)	Speed (km/hr)	Climb (m/min)	Fuel Burn (kg/hr)
Full power cruise	204	54	259	0	22.9
Economy cruise	221	54	244	0	18.8
Climb	221	109	147	372	38.1

Table 6: Preliminary Design Values

The following questions were posed to the end-user, and the following answers received.

1. When you were selecting an aircraft kit, what was the range of advertised cruise speeds of the aircraft (at 75% power, 8000 ft.) in which you were interested? *Answer: 120 to 200 mph*
2. Approximately how much additional money would you be willing to spend for the aircraft in order to increase the cruise speed of your airplane by 5 mph above the average of the figures in question 1? *Answer: $350*
3. When you were selecting an aircraft kit, what was the range of advertised ranges of the aircraft in which you were interested (at 65% power, 8000 ft.)? *Answer: 500 to 1000 miles*
4. Approximately how much money would you be willing to spend in order to increase the range of your airplane by 50 miles over the average of the figures in question 3? *Answer: $300*
5. When you were selecting an aircraft kit, what was the range of advertised climb rates of the aircraft in which you were interested? *Answer: 700 to 1800 fpm*
6. Approximately how much money would you be willing to spend in order to increase the climb rate of your airplane by 100 fpm above the average of the figures in question 1? *Answer: $250*

For all calculations, it was assumed that the aircraft would have a lifespan of 20 years, be flown 200 hours per year, with 50% of those hours at economy cruise and the remaining 50% at high-speed cruise. Hours spent in climb are considered negligible. An interest rate of 8% was used.

---

[18] These values are from a simple aircraft performance model. The program itself is in Appendix B. The values computed for speed and range are very close to manufacturer's test data. However, climb rates calculated by the program appear approximately 300 fpm too low. For the purpose of the example, values from the model will be used as is.

[19] Although exergy and performance calculations for economy cruise were performed with half fuel so as to approximate an average, range calculations were performed with the flight beginning with full fuel (109 kg)

(b) Cost-benefit Curves

With this information linear functions for the three desiderata were created, and are shown in Figures 8-10.

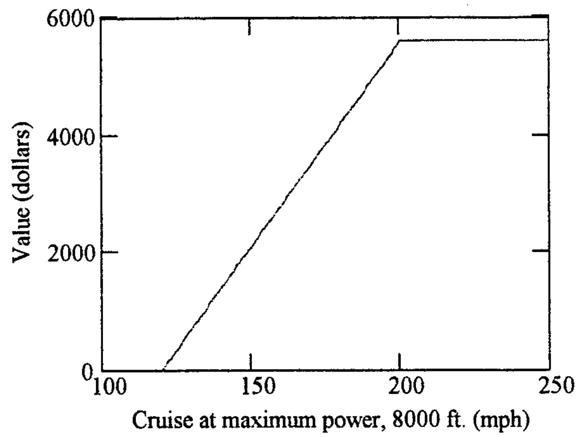


Figure 8: Value versus Cruise Speed

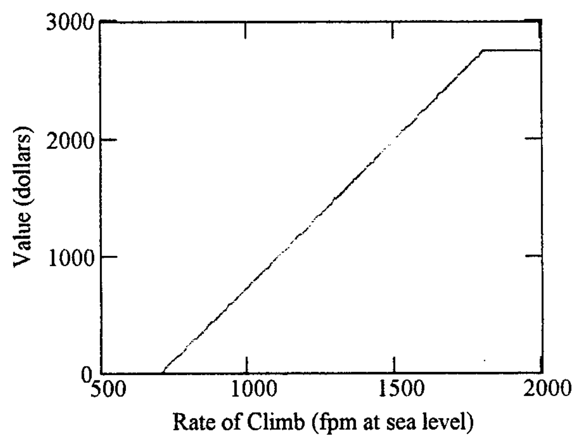


Figure 9: Value versus Rate of Climb

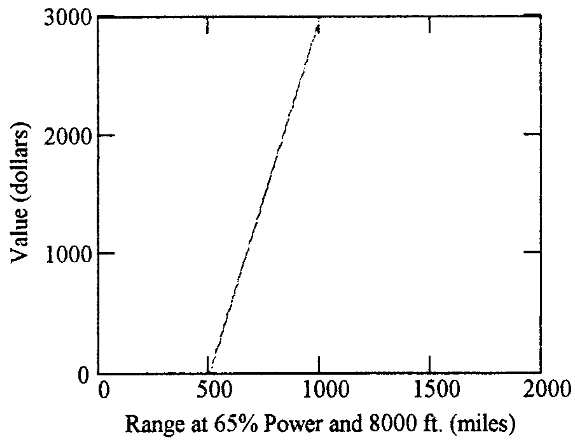


Figure 10: Value versus Range

(c) The Overall Objective Function

The combination of the three cost-benefit curves allows an overall objective function to be formed. The value of each of the desiderata may be written as:

$$Z_{V_{cruise}} = 0\text{dol} \in 120\text{mph} > V_{cruise}$$

$$Z_{V_{cruise}} = \left(V_{cruise} - 120\text{mph}\right) 70 \frac{\text{dol}}{\text{mph}} \in 120\text{mph} \le V_{cruise} \le 200\text{mph} \tag{44}$$

$$Z_{V_{cruise}} = 5600\text{dol} \in 200\text{mph} < V_{cruise}$$

$$Z_{RC} = 0\text{dol} \in 700\text{fpm} > RC$$

$$Z_{RC} = \left(RC - 700\text{mph}\right) 2.5 \frac{\text{dol}}{\text{fpm}} \in 700\text{fpm} \le RC \le 1800\text{fpm} \tag{45}$$

$$Z_{RC} = 2750\text{dol} \in 1800\text{fpm} < RC$$

$$Z_R = 0\text{dol} \in 500\text{mile} > R$$

$$Z_R = \left(R - 500\text{mph}\right) 6 \frac{\text{dol}}{\text{mile}} \in 500\text{mile} \le R \le 1000\text{mile} \tag{46}$$

$$Z_R = 3000\text{dol} \in 1000\text{mile} < R$$

Here V_C represents cruise speed at 75% power and 8000 ft., RC the rate of climb at seal level and R the range at 65% power and 8000 ft.

Now the objective function, to be maximized, may be written as

$$J = Z_{V_C} + Z_{RC} + Z_R - Z_{Capital} + Z_{mean} \tag{47}$$

It is desired to maximize the value of the aircraft's performance minus the cost of the aircraft. Z_{mean} is the mean value of a light aircraft to the user.

Although the questioning of the user did not determine Z_{mean}, (47) is adequate for making decisions between candidate systems and for preliminary design, as Z_{mean} is a constant which will cancel in compressions or optimization.

(d) Exergy Flow Diagrams

Figures 11, 12 and 13 show the exergy flows and per unit exergy costs in a light aircraft for full power cruise, economy cruise and climb. Exergy destructions are also given.

These figures were developed by (i) writing an exergy balance on each device, (ii) writing cost balances on each device and adding auxiliary equations.[20] Additionally, it is necessary to fix a unit cost of exergy for either thrust or fuel; this was done either equation (9) or with fuel cost and equation (10), depending on the flight conditions.

Additionally, engine and propeller performance relations were used. These were in the form of a user's manual for the Lycoming engine and a computerized propeller map supplied by Hartzell. Aircraft performance was estimated from airfoil and manufacturer's test data.

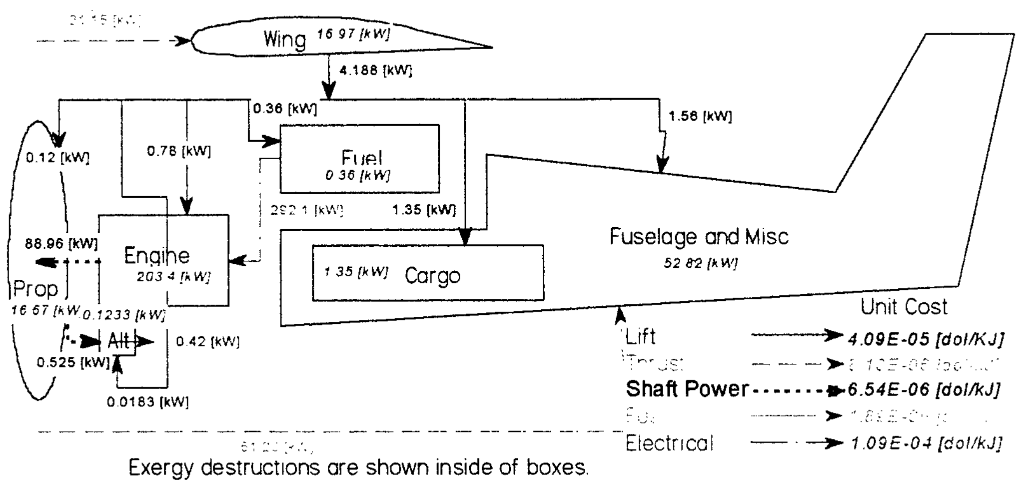


Figure 11: Exergy Flows and Unit Costs, 75% Power, 8000 ft.

[20] In the interest of conserving space, the systems of equations are listed in Appendix B. A good explanation of the methods employed to develop the equations, along with simple examples, is found on pages 3-50 of *Efficiency and Costing*, 1983, edited by R.A. Gaggioli. An additional valuable reference is Bejan et al., 1996.

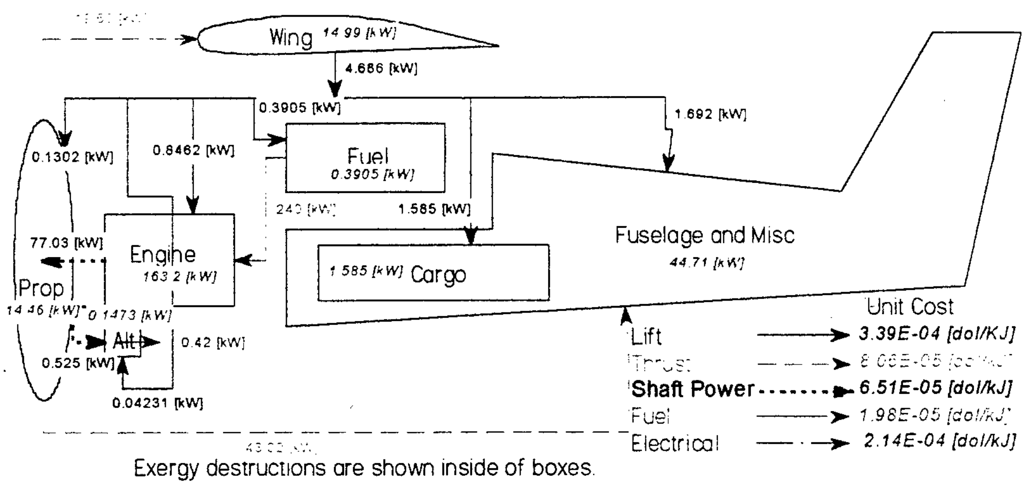


Figure 12: Exergy Flows and Unit Costs, 65% Power, 8000 ft.

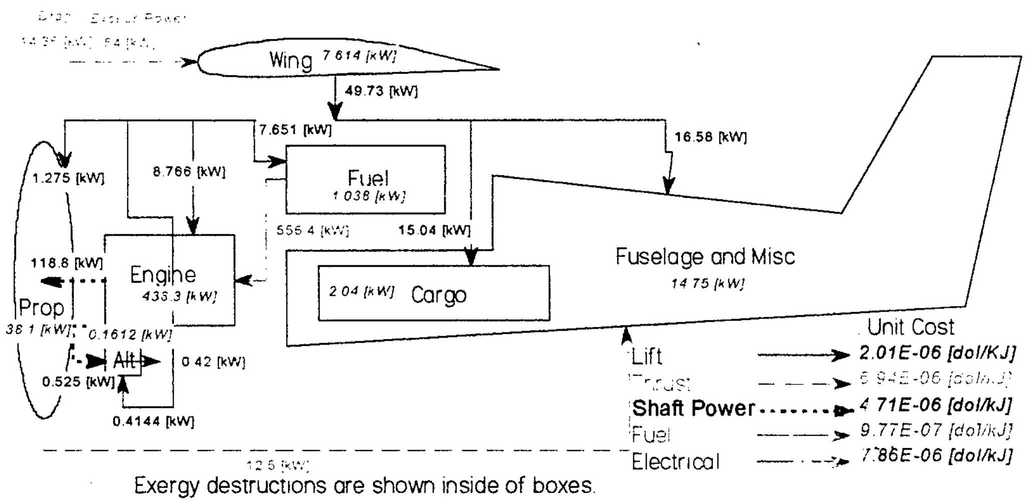


Figure 13: Exergy Flows and Unit Costs, Maximum Climb at Sea Level

(e) Application to Subsystem Design/Selection: Alternator Selection

Here it will be decided whether it is better to purchase a "standard" alternator or a lightweight model. As no efficiency data is available, both will be assumed equally efficient. The standard alternator has an initial cost of $294 and a mass of 5.9-kg (13-lbm). The alternative costs $450, but has a mass of only 2.7-kg (6-lbm). The selection will first be made using the foregoing decomposition process. The results will be checked by application of the overall objective function. (The relative simplicity of modeling a light aircraft makes this application straightforward.)

To make this decision via decomposition, equation (11) is used. Terms for shaft power are unnecessary, as the power inputs to both units are the same. The cost of switching from the standard to the lightweight alternator will be given by

$$\Delta J = Z_{lightweight} - Z_{standard} + \sum_i W_i c_{mi} \left(m_{lightweight} - m_{standard} \right) \tag{48}$$

The lifespan of 20 years, with an interest rate of 8% and 200 operating hours per year yields 0.229 and 0.150 dollars/hr for values of Z for the lightweight and standard alternators, respectively.

Using the performance values from Table 1, along with the information from Figures 8-10[21], yields weighting factors of 0.565 for high-speed cruise, 0.283 for range and 0.153 for climb rate.

Equation (20) is used to find the costs of mass, with the cost of lift taken from the preceding exergy flow diagrams. This yields marginal mass costs of $4.17\cdot10^{-4}$, $3.33\cdot10^{-3}$ and $5.32\cdot10^{-4}$ dollars/kg·hr for full power cruise, economy cruise and rate of climb, respectively.

Finally, evaluating equation (48) with these values yields an increase in cost for the aircraft of 0.0608 dollars/hr, or a present value of 118 dollars, associated with the lightweight alternator. Therefore it is not desirable to use the lightweight alternator over the standard model.

The foregoing conclusion was checked by direct application of the overall aircraft objective function. Evaluating the aircraft performance by simulating the aircraft with the model used for design point calculations, along with the overall objective function in equation (47) yields an increase of cost of 121 dollars for the lightweight alternator. In terms of overall aircraft cost, or even alternator cost, the difference between the values found through decomposition and through overall simulation is insignificant.

(f) Application to a Propulsion System: Engine Selection

The 160-hp engine is the "standard" engine, chosen most widely by builders. The 180-hp engine will produce gains in full power cruise and climb rate, but at the expense of increased weight, decreased range, increased fuel consumption and increased capital cost. (Table 4)

Engine (hp)	Cost (k dollars)	Mass (kg)	Full Power Cruise (km/hr)/ Fuel Burn (kg/hr)	Fuel Burn @ 244 km/hr (kg/hr)	Rate of Climb (m/min)
160	22.3	116	259/22.9	18.8	372
180	26.2	122	271/29.9	21.8	442

Table 7: Performance with different engines

This information may be applied directly to the overall objective function, which results in decrease of profit (present value) of 10,800 dollars.

Alternatively, equation (12) may be employed in the form

[21] The slopes of these figures yield the values of \$350/5-mph, \$300/50-miles and \$250/100-fpm.

$$\Delta J = \sum_i W_i \begin{pmatrix} \sum_{i,j} v_{power,i}\left(\dot{X}_{power,i,180} - \dot{X}_{power,i,160}\right) \\ -c_{mi}\left(m_{180} - m_{160}\right) \end{pmatrix} - \left(Z_{180} - Z_{160}\right) - \sum_i T_i \sum_i c_{fuel_i} \dot{m}_{fuel_i} \tag{49}$$

The unit costs of power (i.e., exergy) may be taken from the exergy flow diagrams and the weighting factors remain the same as the previous examples. The time factors are 0.5 for full power cruise, 0.5 for economy cruise and 0 for climb. Fuel costs are calculated on a per unit mass basis here, as fuel is the raw energy source. The value used is 0.735 dollars/kg. (Exergy costing with the fuel exergy costs assigned would yield the same values.) The cost of fuel burn to range must be added to the cost of fuel for evaluation of range at economy cruise, as per equation (10), increasing the per kilogram cost of fuel to 0.912 dollars.

Evaluation of equation (49), shows a decrease of present value profit of 11,400 dollars for the 180-hp engine. Although this differs from the 10,800-dollar non-decomposed value by 5.5%, the same decision would be reached. The error likely results from the large shift from the preliminary design values, shifts in engine power of 12%, (engine) weight of 6% and fuel consumption of 14%. In such cases iteration in the exergy costing step would improve results if necessary.

IX. CONCLUSIONS

A general outline of the design process has been laid out in a manner that is applicable to the design of thermal systems in general, and vehicular-based energy systems in particular. In support of this general process, means of constructing an overall objective function have been discussed. One area in the thermal design process that has been particularly lacking is the detailed design process. Much work has been accomplished in the area of generic optimization; this leaves off before an actual working system is penned. A thermoeconomic approach to detailed design is presented here. Finally, notes are made (i) on the selection of a "dead state" for exergy analysis (ii) as to when marginal exergy costs should be used.

(a) The Overall Objective Function

One method was outlined for finding an overall objective function. In any case, the subjectivity of the customers' desires for a product must be objectified for optimization to occur. Without an overall objective function for, for example, an aircraft, it is impossible to develop objective functions to guide engineers in the design of energy systems, subsystems and conversion devices.

(b) Detailed Design

Second-law based decomposition strategies do not only aid in mathematical, "generic" optimization, but allow detailed design to proceed in a logical, efficient manner. It allows objective functions to be written for subsystems and devices. Without these objective functions, rational decisions may not be made during the design of these subsystems and devices *without recourse to the simulation of the whole system or vehicle.* Such simulation is time and resource consuming. Within this dissertation, objective functions for aircraft energy systems were developed in detail.

So as to apply exergy concepts to aircraft, exergy flow diagrams were developed for a single-engine general aviation aircraft. To do this it was necessary to derive an expression for the exergy of lift. These exergy flow diagrams, along with thermoeconomics were used to place

costs on weight, and to add costs to fuel to account for range as a desideratum. It was also shown how to develop weighting factors for different realms of performance.

Although the objective functions for detailed design as presented here were developed specifically for aircraft design, they are readily extensible to other vehicles as well as stationary systems.

(c) The "Dead State"

Before choosing a "dead state" for exergy calculations, the constraints on the system must be considered. For one such set of constraints an example of the calculation of the dead state, using the available energy theory of Gibbs, was made.

The calculation was made here for a system with fixed constraints independent of the environment; in general, these constraints will vary with time and may be dependent on the environment. This is often the result of changing environmental conditions, such as varying weather, or the system itself moving (such as an aircraft in climb). If this is the case, integration of exergy calculations over time or weighting of calculations with different dead states may be necessary.

A system with constraints independent of the environment need not be a fixed plant, although the example in this dissertation was. The equilibrium temperature as defined through available energy theory has applicability to vehicle design as well. An aircraft subsystem may be operating between two thermal "sinks", for example, the avionics and the fuel, and never "see" the external environment.

The *fundamental* conclusion is this. The available energy *concept* introduced by Gibbs is more fundamental than the exergy concept, which is a special case of available energy of a "body and a (large) medium" in Gibbs terminology. As shown by Gaggioli (1995, 1998), in any case (i) the equations representing contributions of subsystems to the overall *available energy* of a system can be derived without reference to any environment; (ii) for each subsystem, that equation includes properties of that subsystem's dead state; (iii) the subsystem's dead state properties are found by minimization of the overall system energy subject to the constraints on each of the subsystems while maintaining overall system entropy. The example presented here represents an application of that theory, and shows an example when that theory is needed, practically.

(d) Marginal Exergy Costs

When the design process moves to detailed design *marginal* exergy costs must be used by the device design teams. These may often be approximated by incremental costs. However, one example was given when the incremental costs would not be proper, a combined-cycle power plant with duct firing. Many times it will be obvious if approximations may be made. If it is not obvious, mathematical checks should be made before the assumptions!

(e) Case Studies

The tools for detailed design, namely the creation of exergy flow diagrams for an aircraft and the development of overall and device objective functions have been demonstrated in a real-world case study of a homebuilt aircraft.

(f) Future Work and Recommendations

Future work should be centered in two areas:

1. The development of the overall objective function for combat aircraft. In this dissertation, methods by which this may be approached have been proposed, but not tested. These proposals should be tested and refined.
2. The device and subsystem objective functions should be applied to past decisions in industry. Ideally, the decisions should have involved computing the effects on the aircraft's performance. This will help validate the methodology. The aircraft would ideally be a supersonic aircraft, so as to ensure that the exergy of lift, as developed here, is applicable to supersonic flight.

X. REFERENCES

1. Bejan, Adrian, Tsatsaronis, G. and Moran, M.J., 1996, *Thermal Design and Optimization*, Wiley Interscience, New York, NY
2. El-Sayed, Y.M. and Evans, R.B., 1970, *ASME Journal of Engineering for Power*, Vol. 92, p.27
3. El-Sayed, Y.M., 1995, "Repowering Second-Law-Based Optimization", AES-Vol. 35, ASME, New York, NY
4. El-Sayed, Y.M., 1996, "A Second-Law-Based Optimization: Part 1 – Methodology", *ASME Journal for Gas Turbines and Power*, Vol. 118, pp. 693-697
5. Evans, R.B. and von Spakovsky, M.R., 1984, *Second Law Aspects of Thermal Design*, ASME Vol. HTD 33, p. 27
6. Gaggioli, R.A., editor, 1983, *Efficiency and Costing*, American Chemical Society, Washington, D.C.
7. Gaggioli, R.A., A.J. Bowman and D.H. Richardson, 1999, "Available Energy: I. Gibbs Revisited", AES-Vol. 39, pp. 285-296, ASME, New York
8. Gaggioli, R.A. and D.M. Paulus, Jr., 1999, "Available Energy: II. Gibbs Extended", AES-Vol. 39, pages 285-296, ASME, New York
9. McCormick, Barnes W., 1995, *Aerodynamics, Aeronautics and Flight Mechanics*, John Wiley and Sons, New York, NY
10. McGhee, Robert J., W.D. Beasley and D.M. Somers, 1977, *Low-Speed Characteristics of a 13-Percent-Thick Airfoil Section Designed for General Aviation Applications*, NASA Technical Memorandum TM X-72697
11. Moran, Michael J., 1989, *Availability Analysis: A Guide to Efficient Energy Use*, ASME Press, New York, NY

APPENDIX A: GLASTAR AIRCRAFT INFORMATION[1]

Performance	125 h.p.	160 h.p.	180 h.p.
Top Speed (TAS at sea level)	156 m.p.h. / 136 kts.	167 m.p.h. / 145 kts.	171 m.p.h. / 149 kts.
Cruise Speed (TAS)			
75% power at 8,000 ft.	151 m.p.h. / 131 kts.	161 m.p.h. / 140 kts.	167 m.p.h. / 145 kts.
65% power at 8,000 ft	140 m.p.h. / 122 kts.	153 m.p.h. / 133 kts.	159 m.p.h. / 138 kts.
Stall Speed (at max. gross)			
No flaps (Vs)		56 m.p.h. / 49 kts.	
Full flaps (Vso)		49 m.p.h. / 43 kts.	
Rate of Climb			
Solo	1,300 f.p.m.	2,075 f.p.m.	2,150 f.p.m.
Max. gross	1,000 f.p.m.	1,390 f.p.m.	1,500 f.p.m.
Range (at 65% power)			
Standard tanks	598 mi. / 520 n.m.	553 mi. / 481 n.m.	436 mi. / 379 n.m.
With auxiliary tanks	1,021 mi. / 888 n.m.	953 mi. / 829 n.m.	762 mi. / 663 n.m.
Fuel Consumption (at 65% power)	5.8 g.p.h.	6.7 g.p.h.	8.5 g.p.h.
Service Ceiling (estimated)	17,000 ft.	20,000 ft.	21,000 ft.

Specifications	
Fuselage Length	
Continental IO-240 (rigged for flight)	22.3 ft.
Continental IO-240 (wings folded)	24.5 ft.
Lycoming engine (rigged for flight)	22.8 ft.
Continental IO-240 (wings folded)	25.0 ft.
Wing Span	
Rigged for flight	35.0 ft.
Wings folded & tail removed	8.0 ft.
Other Wing Data	
Airfoil	NASA GA(W)-2
Area	128.0 sq. ft.
Aspect ratio	9.6
Wing loading (at max. gross)	15.3 lbs. per sq. ft.
Structural limit loads (at max. gross)	+3.8 / -1.5 Gs
Maximum Height	

[1] Information is from the former Stoddard-Hamilton company's website. Performance data is from company-built aircraft.

Tricycle (on gear)	9.1 ft.
Tricycle (wings folded)	7.1 ft.
Taildragger	6.9 ft.
Cabin Dimensions	
Width (at hips)	44.0 in.
Width (at shoulders)	46 in.
Door width	37.0 in.
Door height	31.5 in.
Baggage space	32.0 cu. ft.
Weights	
Maximum gross weight (on wheels)	1,960 lbs.
Maximum gross weight (on floats)	2,100 lbs.
Empty weight (typical)	1,200 lbs.
Useful load (typical)	760 lbs.
Full-fuel payload (standard tanks)	520 lbs.
Maximum baggage capacity	250 lbs.
Fuel Capacity (usable)	
Fuel cells	40.0 gals.

APPENDIX B. EES MODELS

B.01 "Beer and Soup" Models

(a) With T_0 Varied

```
"Refrigeration Cycle"

"Compressor"
m_dot*(h_VI-h_VII)+W_dot_V=0 "[kW]"
m_dot*(h_VI-h_VII_s)+W_dot_V_s=0 "[kW]"
m_dot*(s_VI-s_VII)+S_PI_V=0 "[kW/K]"
m_dot*(s_VI-s_VII_s)=0 "[kW/K]"
X_dot_VI-X_dot_VII+W_dot_V-X_dot_d_V=0 "[kW]"
h_VI=enthalpy(r114, p=p_VI, s=s_VI) "[kJ/kg]"
h_VII=enthalpy(r114, p=p_VII, s=s_VII) "[kJ/kg]"
h_VII_s=enthalpy(r114, p=p_VII, s=s_VII_s) "[kJ/kg]"
X_dot_VI=m_dot*(h_VI-h_0-T_0*(s_VI-S_0)) "[kW]"
X_dot_VII=m_dot*(h_VII-h_0-T_0*(s_VII-S_0)) "[kW]"
eta_V=W_dot_V_s/W_dot_V

"Condenser"
m_dot*(h_VII-h_TI)-Q_dot_C=0 "[kW]"
m_dot*(s_VII-s_TI)-S_tau_c+S_PI_C=0 "[kW/K]"
X_dot_VII-X_dot_C-X_dot_TI-X_dot_d_C=0 "[kW]"
X_dot_TI=m_dot*(h_TI-h_0-T_0*(s_TI-S_0)) "[kW]"
h_TI=enthalpy(r114, p=p_TI, s=s_TI) "[kJ/kg]"
S_tau_C=Q_dot_C/T_soup "[kW/K]"
X_dot_C=(T_soup-T_0)*S_tau_C "[kW]"

"Turbine"
m_dot*(h_TI-h_TII)-W_dot_T=0 "[kW]"
m_dot*(h_TI-h_TII_s)-W_dot_T_s=0 "[kW]"
m_dot*(s_TI-s_TII)+S_PI_T=0 "[kW/K]"
m_dot*(s_TI-s_TII_s)=0 "[kW/K]"
X_dot_TI-X_dot_TII-W_dot_T-X_dot_d_T=0 "[kW]"
h_TII=enthalpy(r114, p=p_TII, s=s_TII) "[kJ/kg]"
h_TII_s=enthalpy(r114, p=p_TII, s=s_TII_s) "[kJ/kg]"
X_dot_TII=m_dot*(h_TII-h_0-T_0*(s_TII-S_0)) "[kW]"
eta_T=W_dot_T/W_dot_T_s

"Evaporator"
m_dot*(h_TII-h_VI)+Q_dot_E=0 "[kW]"
m_dot*(s_TII-s_VI)+S_tau_E+S_PI_E=0 "[kW/K]"
X_dot_TII-X_dot_VI--X_dot_E-X_dot_d_E=0 "[kW]"
S_tau_E=Q_dot_E/T_beer "[kW/K]"
X_dot_E=-(T_beer-T_0)*S_tau_E "[kW]"

"Equilibrium"
s_0=entropy(r114, T=T_0, x=0) "[kJ/kg-K]"
h_0=enthalpy(r114, T=T_0, x=0) "[kJ/kg]"
```

```
c_p=4.2 "[kJ/kg-K]"
m_dot_beer*c_p*(T_beer_I-T_beer_II)-Q_dot_E=0
m_dot_soup*c_p*(T_soup_I-T_soup_II)+Q_dot_C=0
T_soup_I=300 "[K]"
T_beer_I=288 "[K]"
T_beer=(T_beer_I-T_beer_II)/ln(T_beer_I/T_beer_II)
T_soup=(T_soup_I-T_soup_II)/ln(T_soup_I/T_soup_II)

"Boundary Conditions"
T_soup=323 "[K]"
T_beer=282 "[K]"
p_VII=pressure(r114, T=T_soup+27, x=0) "[kPa]"
p_TII=pressure(r114, T=T_beer-27, x=0) "[kPa]"
p_TI=p_VII "[kPa]"
p_VI=p_TII "[kPa]"
T_sat_C=temperature(r114, p=p_VII, x=0) "[K]"
T_sat_E=temperature(r114, p=p_TII, x=0) "[K]"
s_VI=entropy(r114, p=p_VI, T=T_VI) "[kJ/kg-K]"
s_TI=entropy(r114, p=p_TI, x=q_TI) "[kJ/kg-K]"
q_TI=0.3
T_VI=T_sat_E+5 "[K]"
eta_T=0.0
eta_V=0.7
{X_dot_E=1.544*22 "[kW]"}
m_dot=88.46 "[kg/s]"

"Calculated"
eta_II=(X_dot_E+X_dot_C)/(W_dot_V-W_dot_T)
T_VII=temperature(r114, p=p_VII, s=s_VII) "[K]"
T_TII=temperature(r114, p=p_TII, s=s_TII) "[K]"
q_TII=quality(r114, p=p_TII, s=s_TII)
q_VII=quality(r114, p=p_VII, s=s_VII)
W_dot_net=W_dot_V-W_dot_T "[kW]"
X_dot_VI_out=-X_dot_VI "[kW]"
X_dot_TII_in=-X_dot_TII "[kW]"

"Costing"
Z_V+c_power*W_dot_V=c_VI*X_dot_VI_out+c_VII*X_dot_VII
Z_C+c_VII*X_dot_VII=c_TI*X_dot_TI+c_X_C*X_dot_C
Z_T+c_TI*X_dot_TI+c_TII*X_dot_TII_in=c_power*W_dot_T
Z_E+c_VI*X_dot_VI_out=c_TII*X_dot_TII_in+c_X_E*X_dot_E
{Z_V_A=25000*.15*(m_dot*convert(kg/s,
lb_m/s))*(p_VII/p_VI)^0.45*(eta_V/(1-eta_V))^0.65 "[$]"
Z_T_A=25000*.32*(m_dot*convert(kg/s,
lb_m/s))*(p_TI/p_TII)^0.5*(eta_T/(1-eta_T))^0.85 "[$]"
Z_C_A=15*15*Q_dot_C*15^(-1)*6^(-.16)*.005^(-.04)
Z_E_A=15*15*Q_dot_E*15^(-1)*6^(-.16)*.005^(-.04)}
Z_V_A=0 "[$]"
Z_T_A=0 "[$]"
Z_C_A=0
Z_E_A=0
Z_V=Z_V_A*.1175/(365.25*24* 3600) "[$/s]"
```

```
Z_T=Z_T_A*.1175/(365.25*24* 3600) "[$/s]"
Z_C=Z_C_A*.1175/(365.25*24* 3600) "[$/s]"
Z_E=Z_E_A*.1175/(365.25*24* 3600) "[$/s]"
C_dot_beer=c_X_E*X_dot_E*convert(1/s,1/hr) "[$/hr]"
C_dot_soup=c_X_C*X_dot_C*convert(1/s,1/hr) "[$/hr]"
c_power=0.07/3600 "[$/kJ]"
c_VII=c_VI
c_TII=c_TI
```

(b) ***With*** T_{eq}

```
"Refrigeration Cycle"

"Compressor"
m_dot*(h_VI-h_VII)+W_dot_V=0 "[kW]"
m_dot*(h_VI-h_VII_s)+W_dot_V_s=0 "[kW]"
m_dot*(s_VI-s_VII)+S_PI_V=0 "[kW/K]"
m_dot*(s_VI-s_VII_s)=0 "[kW/K]"
X_dot_VI-X_dot_VII+W_dot_V-X_dot_d_V=0 "[kW]"
h_VI=enthalpy(r114, p=p_VI, s=s_VI) "[kJ/kg]"
h_VII=enthalpy(r114, p=p_VII, s=s_VII) "[kJ/kg]"
h_VII_s=enthalpy(r114, p=p_VII, s=s_VII_s) "[kJ/kg]"
X_dot_VI=m_dot*(h_VI-h_0-T_0*(s_VI-S_0)) "[kW]"
X_dot_VII=m_dot*(h_VII-h_0-T_0*(s_VII-S_0)) "[kW]"
eta_V=W_dot_V_s/W_dot_V

"Condenser"
m_dot*(h_VII-h_TI)-Q_dot_C=0 "[kW]"
m_dot*(s_VII-s_TI)-S_tau_c+S_PI_C=0 "[kW/K]"
X_dot_VII-X_dot_C-X_dot_TI-X_dot_d_C=0 "[kW]"
X_dot_TI=m_dot*(h_TI-h_0-T_0*(s_TI-S_0)) "[kW]"
h_TI=enthalpy(r114, p=p_TI, s=s_TI) "[kJ/kg]"
S_tau_C=Q_dot_C/T_soup "[kW/K]"
X_dot_C=(T_soup-T_0)*S_tau_C "[kW]"

"Turbine"
m_dot*(h_TI-h_TII)-W_dot_T=0 "[kW]"
m_dot*(h_TI-h_TII_s)-W_dot_T_s=0 "[kW]"
m_dot*(s_TI-s_TII)+S_PI_T=0 "[kW/K]"
m_dot*(s_TI-s_TII_s)=0 "[kW/K]"
X_dot_TI-X_dot_TII-W_dot_T-X_dot_d_T=0 "[kW]"
h_TII=enthalpy(r114, p=p_TII, s=s_TII) "[kJ/kg]"
h_TII_s=enthalpy(r114, p=p_TII, s=s_TII_s) "[kJ/kg]"
X_dot_TII=m_dot*(h_TII-h_0-T_0*(s_TII-S_0)) "[kW]"
eta_T=W_dot_T/W_dot_T_s

"Evaporator"
m_dot*(h_TII-h_VI)+Q_dot_E=0 "[kW]"
m_dot*(s_TII-s_VI)+S_tau_E+S_PI_E=0 "[kW/K]"
X_dot_TII-X_dot_VI--X_dot_E-X_dot_d_E=0 "[kW]"
S_tau_E=Q_dot_E/T_beer "[kW/K]"
```

```
X_dot_E=-(T_beer-T_0)*S_tau_E "[kW]"

"Equilibrium"
s_0=entropy(r114, T=T_0, x=0) "[kJ/kg-K]"
h_0=enthalpy(r114, T=T_0, x=0) "[kJ/kg]"
c_p=4.2 "[kJ/kg-K]"
m_dot_beer*c_p*(T_beer_I-T_beer_II)-Q_dot_E=0
m_dot_soup*c_p*(T_soup_I-T_soup_II)+Q_dot_C=0
T_soup_I=300 "[K]"
T_beer_I=288 "[K]"
T_beer=(T_beer_I-T_beer_II)/ln(T_beer_I/T_beer_II)
T_soup=(T_soup_I-T_soup_II)/ln(T_soup_I/T_soup_II)
T_0_I=T_soup_I*(T_soup_I/T_beer_I)^((-
m_dot_beer*c_p)/(m_dot_beer*c_p+m_dot_soup*c_p)) "[K]"
T_0_II=T_soup_II*(T_soup_II/T_beer_II)^((-
m_dot_beer*c_p)/(m_dot_beer*c_p+m_dot_soup*c_p)) "[K]"
T_0=(T_0_I-T_0_II)/ln(T_0_I/T_0_II)

"Boundary Conditions"
T_soup=323 "[K]"
T_beer=282 "[K]"
p_VII=pressure(r114, T=T_soup+27, x=0) "[kPa]"
p_TII=pressure(r114, T=T_beer-27, x=0) "[kPa]"
p_TI=p_VII "[kPa]"
p_VI=p_TII "[kPa]"
T_sat_C=temperature(r114, p=p_VII, x=0) "[K]"
T_sat_E=temperature(r114, p=p_TII, x=0) "[K]"
s_VI=entropy(r114, p=p_VI, T=T_VI) "[kJ/kg-K]"
s_TI=entropy(r114, p=p_TI, x=q_TI) "[kJ/kg-K]"
q_TI=0.3
T_VI=T_sat_E+5 "[K]"
eta_T=0.0
eta_V=0.7
m_dot=88.46 "[kg/s]"

"Calculated"
eta_II=(X_dot_E+X_dot_C)/(W_dot_V-W_dot_T)
T_VII=temperature(r114, p=p_VII, s=s_VII) "[K]"
T_TII=temperature(r114, p=p_TII, s=s_TII) "[K]"
q_TII=quality(r114, p=p_TII, s=s_TII)
q_VII=quality(r114, p=p_VII, s=s_VII)
W_dot_net=W_dot_V-W_dot_T "[kW]"
X_dot_VI_out=-X_dot_VI "[kW]"
X_dot_TII_in=-X_dot_TII "[kW]"

"Costing"
Z_V+c_power*W_dot_V=c_VI*X_dot_VI_out+c_VII*X_dot_VII
Z_C+c_VII*X_dot_VII=c_TI*X_dot_TI+c_X_C*X_dot_C
Z_T+c_TI*X_dot_TI+c_TII*X_dot_TII_in=c_power*W_dot_T
Z_E+c_VI*X_dot_VI_out=c_TII*X_dot_TII_in+c_X_E*X_dot_E

Z_V_A=0 "[$]"
```

```
Z_T_A=0 "[$]"
Z_C_A=0
Z_E_A=0
Z_V=Z_V_A*.1175/(365.25*24* 3600) "[$/s]"
Z_T=Z_T_A*.1175/(365.25*24* 3600) "[$/s]"
Z_C=Z_C_A*.1175/(365.25*24* 3600) "[$/s]"
Z_E=Z_E_A*.1175/(365.25*24* 3600) "[$/s]"
C_dot_beer=c_X_E*X_dot_E*convert(1/s,1/hr) "[$/hr]"
C_dot_soup=c_X_C*X_dot_C*convert(1/s,1/hr) "[$/hr]"
c_power=0.07/3600 "[$/kJ]"
c_VII=c_VI
c_TII=c_TI
```

B.02 Aircraft Models

(a) Cruise, 75% Power, 8000 ft.

```
"Light A/C Exergy Balance and Costing, Cruise"

"Exergy Balances"
"Fuel:" X_dot_L_F-X_dot_d_F=0 "[kW]"
"Engine" X_dot_Ch_F-X_dot_P_P-X_dot_P_A-X_dot_Exh+X_dot_L_E-
   X_dot_d_E=0 "[kW]"
"Propeller" X_dot_P_P+X_dot_L_P-X_dot_T-X_dot_d_P=0 "[kW]"
"Alternator" X_dot_P_A+X_dot_L_A-X_dot_elect-X_dot_d_A=0
   "[kW]"
"Wing" X_dot_T_W-X_dot_L_net-X_dot_d_W=0 "[kW]"
"Fuselage etc.:" X_dot_T_M+X_dot_L_M-X_dot_d_M=0 "[kW]"
"Cargo:" X_dot_L_C-X_dot_d_C=0 "[kW]"

"Auxiliary Relations"
X_dot_L_net=X_dot_L-X_dot_L_W
X_dot_L_F=m_F/m_G*X_dot_L
X_dot_L_E=m_E/m_G*X_dot_L
X_dot_L_P=m_P/m_G*X_dot_L
X_dot_L_A=m_A/m_G*X_dot_L
X_dot_L_W=m_W/m_G*X_dot_L
X_dot_L_M=m_M/m_G*X_dot_L
X_dot_L_C=m_C/m_G*X_dot_L
X_dot_T=X_dot_T_W+X_dot_T_M
X_dot_T=eta_P*X_dot_P_P
X_dot_elect=eta_A*X_dot_P_A
X_dot_elect=volt*i*convert(W, kW)
X_dot_L=m_G*g*(RC-RC_0)*convert(W, kW)
X_dot_T_W=V*rho_air*V^2/2*C_D*A_wing*convert(W,kW)
X_dot_Ch_F=m_dot_F*X_Ch_F
X_dot_T_M=V*rho_air*V^2/2*A_equiv_M*convert(W,kW)
P_req=X_dot_T_W+X_dot_T_M "[kW]"
P_ex=X_dot_T-P_req "[kW]"
RC_0=-rho_air*V^3*C_L^2*A_wing/(2*pi*a)/(m_G*g)

"Boundary Conditions"
```

```
"Masses"
m_F=120*convert(lb_m, kg) "[kg]"
m_E=260*convert(lb_m, kg) "[kg]"
m_P=40*convert(lb_m, kg) "[kg]" "Includes propeller &
   governor"
m_A=6.1*convert(lb_m, kg) "[kg]"
m_W=400*convert(lb_m, kg) "[kg]"
m_C=450*convert(lb_m, kg) "[kg]"
m_M=520*convert(lb_m, kg) "[kg]"
m_G-m_F-m_E-m_P-m_A-m_W-m_C-m_M=0 "[kg]"

"A/C Geometry"
A_wing=128*convert(ft^2, m^2) "[m^2]"
"Aspect Ratio: " a=9.6

"Equipment Performance"
eta_P=0.814
eta_A=0.8

"A/C Performance"
"Speed: " {V=161*convert(mph, m/s)} "[m/s]"
"Engine Power:" X_dot_P_A+X_dot_P_P=120*convert(hp, kW) "[kW]"
"Fuel Flow:"  m_dot_F=8.4*6*convert(lb_m/hr, kg/s) "[kg/s]"
"Current Draw: " i=30 "[amp]"
"Voltage:" volt=14 "[volt]"
{C_D=0.0075}
k=0.0038
C_D_min=0.007
e=1/(1+delta+k*pi*a)
C_D=C_D_min+C_L^2/(pi*a*e)
delta=0.17
C_L=C_l_alpha*(a/(a+(2*(a+4)/(a+2))))
A_equiv_M=0.2998
V_mph=V*convert(m/s, mph) "[mph]"
RC=0

"Misc"
X_Ch_F=46000 "[kJ/kg]"
X_dot_Exh=0 "[kW]" "Lump in with engine destruction"
rho_air=0.75*density(air, p=101, T=288)
rho_air*V^2/2*C_L*A_wing=m_G*g
g=9.8 "[m/s^2]"


"Costing"
"Fuel:" c_F_total*X_dot_Ch_F=c_F*m_dot_F+c_Li*X_dot_L_F
Z_dot_F=c_F*m_dot_F*convert(1/s,1/hr)
Z_dot_F_total=c_F_total*X_dot_Ch_F*convert(1/s,1/hr)
"Engine" c_F_total*X_dot_Ch_F-c_P*X_dot_P_P-
   c_P*X_dot_P_A+c_Li*X_dot_L_E+Z_E=0
Z_dot_P_P=c_P*X_dot_P_P*convert(1/s,1/hr)
Z_dot_P_A=c_P*X_dot_P_A*convert(1/s,1/hr)
```

```
Z_dot_L_E=c_Li*X_dot_L_E*convert(1/s,1/hr)
"Propeller" c_P*X_dot_P_P+c_Li*X_dot_L_P-c_T*X_dot_T+Z_P=0
Z_dot_T=c_T*X_dot_T*convert(1/s,1/hr)
Z_dot_L_P=c_Li*X_dot_L_P*convert(1/s,1/hr)
"Alternator" c_P*X_dot_P_A+c_Li*X_dot_L_A-
   c_elect*X_dot_elect+Z_A=0
Z_dot_L_A=c_Li*X_dot_L_A*convert(1/s,1/hr)
Z_dot_elect=c_elect*X_dot_elect*convert(1/s,1/hr)
"Wing" c_T*X_dot_T_W-c_Li*X_dot_L_net+Z_W=0
Z_dot_T_W=c_T*X_dot_T_W*convert(1/s,1/hr)
"Fuselage etc.:" c_T*X_dot_T_M+c_Li*X_dot_L_M+Z_M=C_dot_M
Z_dot_T_M=c_T*X_dot_T_M*convert(1/s,1/hr)
Z_dot_T_M=Z_S
Z_dot_L_M=c_Li*X_dot_L_M*convert(1/s,1/hr)
"Cargo:" c_Li*X_dot_L_C=C_dot_C
Z_dot_L_C=c_Li*X_dot_L_C*convert(1/s,1/hr)
"Capital Costs"
c_T=8.1e-6
Z_E=0
Z_P=0
Z_A=4.159e-5
Z_W=0
Z_M=0
```

(b) Cruise, 65% Power, 8000 ft.

```
"Light A/C Exergy Balance and Costing, Cruise"

"Exergy Balances"
"Fuel:" X_dot_L_F-X_dot_d_F=0 "[kW]"
"Engine" X_dot_Ch_F-X_dot_P_P-X_dot_P_A-X_dot_Exh+X_dot_L_E-
   X_dot_d_E=0 "[kW]"
"Propeller" X_dot_P_P+X_dot_L_P-X_dot_T-X_dot_d_P=0 "[kW]"
"Alternator" X_dot_P_A+X_dot_L_A-X_dot_elect-X_dot_d_A=0
   "[kW]"
"Wing" X_dot_T_W-X_dot_L_net-X_dot_d_W=0 "[kW]"
"Fuselage etc.:" X_dot_T_M+X_dot_L_M-X_dot_d_M=0 "[kW]"
"Cargo:" X_dot_L_C-X_dot_d_C=0 "[kW]"

"Auxiliary Relations"
X_dot_L_net=X_dot_L-X_dot_L_W
X_dot_L_F=m_F/m_G*X_dot_L
X_dot_L_E=m_E/m_G*X_dot_L
X_dot_L_P=m_P/m_G*X_dot_L
X_dot_L_A=m_A/m_G*X_dot_L
X_dot_L_W=m_W/m_G*X_dot_L
X_dot_L_M=m_M/m_G*X_dot_L
X_dot_L_C=m_C/m_G*X_dot_L
X_dot_T=X_dot_T_W+X_dot_T_M
X_dot_T=eta_P*X_dot_P_P
X_dot_elect=eta_A*X_dot_P_A
X_dot_elect=volt*i*convert(W, kW)
```

```
X_dot_L=m_G*g*(RC-RC_0)*convert(W, kW)
X_dot_T_W=V*rho_air*V^2/2*C_D*A_wing*convert(W,kW)
X_dot_Ch_F=m_dot_F*X_Ch_F
X_dot_T_M=V*rho_air*V^2/2*A_equiv_M*convert(W,kW)
P_req=X_dot_T_W+X_dot_T_M "[kW]"
P_ex=X_dot_T-P_req "[kW]"
RC_0=-rho_air*V^3*C_L^2*A_wing/(2*pi*a)/(m_G*g)

"Boundary Conditions"
"Masses"
m_F=120*convert(lb_m, kg) "[kg]"
m_E=260*convert(lb_m, kg) "[kg]"
m_P=40*convert(lb_m, kg) "[kg]" "Includes propeller &
   governor"
m_A=13*convert(lb_m, kg) "[kg]"
m_W=400*convert(lb_m, kg) "[kg]"
m_C=220.9 "[kg]"
m_M=520*convert(lb_m, kg) "[kg]"
m_G-m_F-m_E-m_P-m_A-m_W-m_C-m_M=0 "[kg]"

"A/C Geometry"
A_wing=128*convert(ft^2, m^2) "[m^2]"
"Aspect Ratio: " a=9.6

"Equipment Performance"
eta_P=0.814
eta_A=0.8

"A/C Performance"
"Speed: "{ V=161*convert(mph, m/s)} "[m/s]"
"Engine Power:" X_dot_P_A+X_dot_P_P=104*convert(hp, kW) "[kW]"
"Fuel Flow:"  m_dot_F=6.9*6*convert(lb_m/hr, kg/s) "[kg/s]"
"Current Draw: " i=30 "[amp]"
"Voltage:" volt=14 "[volt]"
k=0.0038
C_D_min=0.007
e=1/(1+delta+k*pi*a)
C_D=C_D_min+C_L^2/(pi*a*e)
delta=0.17
C_L=C_l_alpha*(a/(a+(2*(a+4)/(a+2))))
A_equiv_M=0.2988
V_mph=V*convert(m/s, mph) "[mph]"
RC=0

"Misc"
X_Ch_F=46000 "[kJ/kg]"
X_dot_Exh=0 "[kW]" "Lump in with engine destruction"
rho_air=0.75*density(air, p=101, T=288)
rho_air*V^2/2*C_L*A_wing=m_G*g
g=9.8 "[m/s^2]"
Ratio_DL=(X_dot_T*convert(kW, W)/V)/(m_G*g)
BSFC=m_dot_F/X_dot_P_P "[kg/kW-s]"
```

```
"Costing"
"Fuel:" c_F_total*X_dot_Ch_F=c_F*m_dot_F+c_Li*X_dot_L_F
Z_dot_F=c_F*m_dot_F*convert(1/s,1/hr)
Z_dot_F_total=c_F_total*X_dot_Ch_F*convert(1/s,1/hr)
"Engine" c_F_total*X_dot_Ch_F-c_P*X_dot_P_P-
   c_P*X_dot_P_A+c_Li*X_dot_L_E+Z_E=0
Z_dot_P_P=c_P*X_dot_P_P*convert(1/s,1/hr)
Z_dot_P_A=c_P*X_dot_P_A*convert(1/s,1/hr)
Z_dot_L_E=c_Li*X_dot_L_E*convert(1/s,1/hr)
"Propeller" c_P*X_dot_P_P+c_Li*X_dot_L_P-c_T*X_dot_T+Z_P=0
Z_dot_T=c_T*X_dot_T*convert(1/s,1/hr)
Z_dot_L_P=c_Li*X_dot_L_P*convert(1/s,1/hr)
"Alternator" c_P*X_dot_P_A+c_Li*X_dot_L_A-
   c_elect*X_dot_elect+Z_A=0
Z_dot_L_A=c_Li*X_dot_L_A*convert(1/s,1/hr)
Z_dot_elect=c_elect*X_dot_elect*convert(1/s,1/hr)
"Wing" c_T*X_dot_T_W-c_Li*X_dot_L_net+Z_W=0
Z_dot_T_W=c_T*X_dot_T_W*convert(1/s,1/hr)
"Fuselage etc.:" c_T*X_dot_T_M+c_Li*X_dot_L_M+Z_M=C_dot_M
Z_dot_T_M=c_T*X_dot_T_M*convert(1/s,1/hr)
Z_dot_T_M=Z_S
Z_dot_L_M=c_Li*X_dot_L_M*convert(1/s,1/hr)
"Cargo:" c_Li*X_dot_L_C=C_dot_C
Z_dot_L_C=c_Li*X_dot_L_C*convert(1/s,1/hr)
"Capital Costs"
c_F=(2+.414)*1/6*2.2046
Z_E=0
Z_P=0
Z_A=4.159e-5
Z_W=0
Z_M=0
```

(c) Climb, Sea Level (Maximize RC as a function of V)

```
"Light A/C Exergy Balance and Costing, Climb"

"Exergy Balances"
"Fuel:" X_dot_L_F-X_dot_d_F=m_F*g*RC*convert(W, kW)   "[kW]"
"Engine" X_dot_Ch_F-X_dot_P_P-X_dot_P_A-X_dot_Exh+X_dot_L_E-
   X_dot_d_E=m_E*g*RC*convert(W, kW) "[kW]"
"Propeller" X_dot_P_P+X_dot_L_P-X_dot_T-
   X_dot_d_P=m_P*g*RC*convert(W, kW)   "[kW]"
"Alternator" X_dot_P_A+X_dot_L_A-X_dot_elect-
   X_dot_d_A=m_A*g*RC*convert(W, kW)   "[kW]"
"Wing" X_dot_T_W-X_dot_L_net-
   X_dot_d_W+P_ex=m_W*g*RC*convert(W, kW)   "[kW]"
"Fuselage etc.:" X_dot_T_M+X_dot_L_M-
   X_dot_d_M=m_M*g*RC*convert(W, kW)   "[kW]"
"Cargo:" X_dot_L_C-X_dot_d_C=m_C*g*RC*convert(W, kW)   "[kW]"

"Auxiliary Relations"
```

```
X_dot_L_net=X_dot_L-X_dot_L_W
X_dot_L_F=m_F/m_G*X_dot_L
X_dot_L_E=m_E/m_G*X_dot_L
X_dot_L_P=m_P/m_G*X_dot_L
X_dot_L_A=m_A/m_G*X_dot_L
X_dot_L_W=m_W/m_G*X_dot_L
X_dot_L_M=m_M/m_G*X_dot_L
X_dot_L_C=m_C/m_G*X_dot_L
X_dot_T=X_dot_T_W+X_dot_T_M+m_G*g*RC*convert(W, kW)
X_dot_T=eta_P*X_dot_P_P
X_dot_elect=eta_A*X_dot_P_A
X_dot_elect=volt*i*convert(W, kW)
X_dot_L=m_G*g*(RC-RC_0)*convert(W, kW)
X_dot_T_W=V*rho_air*V^2/2*C_D*A_wing*convert(W,kW)
X_dot_Ch_F=m_dot_F*X_Ch_F
X_dot_T_M=V*rho_air*V^2/2*A_equiv_M*convert(W,kW)
P_req=X_dot_T_W+X_dot_T_M "[kW]"
P_ex=X_dot_T-P_req "[kW]"
RC_0=-rho_air*V^3*C_L^2*A_wing/(2*pi*a)/(m_G*g)
P_req=X_dot_T_W+X_dot_T_M "[kW]"
P_ex=X_dot_T-P_req "[kW]"

"Boundary Conditions"
"Masses"
m_G=1960*convert(lb_m, kg) "[kg]" "Gross weight"
m_F=240*convert(lb_m, kg) "[kg]"
m_E=275*convert(lb_m, kg) "[kg]"
m_P=40*convert(lb_m, kg) "[kg]" "Includes propeller &
   governor"
m_A=13*convert(lb_m, kg) "[kg]"
m_W=400*convert(lb_m, kg) "[kg]"
m_G-m_F-m_E-m_P-m_A-m_W-m_C=m_M "[kg]"
m_M=235.9 "[kg]"

"A/C Geometry"
A_wing=128*convert(ft^2, m^2) "[m^2]"
"Aspect Ratio: " a=9.6

"Equipment Performance"
eta_P=0.197+8.25e-3*(V_mph/1.15)-2.723e-5*(V_mph/1.15)^2
eta_A=0.8

"A/C Performance"
"Speed: " V_mph=V*convert(m/s, mph) "[mph]"
"Engine Power:" X_dot_P_A+X_dot_P_P=160*convert(hp, kW) "[kW]"
"Fuel Flow:"  m_dot_F=16*6*convert(lb_m/hr, kg/s) "[kg/s]"
"Current Draw: " i=30 "[amp]"
"Voltage:" volt=14 "[volt]"
"Climb Rate" RC_fpm=RC*convert(m/s, ft/min) "[ft/min]"
k=0.0038
C_D_min=0.007
e=1/(1+delta+k*pi*a)
```

```
C_D=C_D_min+C_L^2/(pi*a*e)
delta=0.17
C_L=C_l_alpha*(a/(a+(2*(a+4)/(a+2))))
A_equiv_M=0.2998 "[m^2]"
Theta_C=arctan(RC/V)

"Misc"
X_Ch_F=46000 "[kJ/kg]"
X_dot_Exh=0 "[kW]" "Lump in with engine destruction"
rho_air=density(air, p=101, T=288)
rho_air*V^2/2*C_L*A_wing=m_G*g
g=9.8 "[m/s^2]"


"Costing"
"Fuel:" c_F_total*X_dot_Ch_F=c_F*m_dot_F+c_Li*X_dot_L_F
Z_dot_F=c_F*m_dot_F*convert(1/s,1/hr)
Z_dot_F_total=c_F_total*X_dot_Ch_F*convert(1/s,1/hr)
"Engine" c_F_total*X_dot_Ch_F-c_P*X_dot_P_P-
   c_P*X_dot_P_A+c_Li*X_dot_L_E+Z_E=0
Z_dot_P_P=c_P*X_dot_P_P*convert(1/s,1/hr)
Z_dot_P_A=c_P*X_dot_P_A*convert(1/s,1/hr)
Z_dot_L_E=c_Li*X_dot_L_E*convert(1/s,1/hr)
"Propeller" c_P*X_dot_P_P+c_Li*X_dot_L_P-c_T*X_dot_T+Z_P=0
Z_dot_T=c_T*X_dot_T*convert(1/s,1/hr)
Z_dot_L_P=c_Li*X_dot_L_P*convert(1/s,1/hr)
"Alternator" c_P*X_dot_P_A+c_Li*X_dot_L_A-
   c_elect*X_dot_elect+Z_A=0
Z_dot_L_A=c_Li*X_dot_L_A*convert(1/s,1/hr)
Z_dot_elect=c_elect*X_dot_elect*convert(1/s,1/hr)
"Wing" c_T*X_dot_T_W-c_Li*X_dot_L_net+Z_W=0
Z_dot_T_W=c_T*X_dot_T_W*convert(1/s,1/hr)
"Fuselage etc.:" c_T*X_dot_T_M+c_Li*X_dot_L_M+Z_M=C_dot_M
Z_dot_T_M=c_T*X_dot_T_M*convert(1/s,1/hr)
Z_dot_L_M=c_Li*X_dot_L_M*convert(1/s,1/hr)
"Cargo:" c_Li*X_dot_L_C=C_dot_C
Z_dot_L_C=c_Li*X_dot_L_C*convert(1/s,1/hr)
"Capital Costs"
Z_E=0
Z_P=0
Z_A=0
Z_W=0
Z_M=0
c_T=6.944e-6 "[dol/kJ]"
```

APPENDIX C. YEAR 2001 PUBLICATIONS

Paulus, D.M. Jr. and R.A. Gaggioli, 2001, "The Exergy of Lift, and Aircraft Exergy Flow Diagrams", ASME AES-Vol 41, ASME, New York (to be submitted to ASME Journal of Energy Resource Technology)

Paulus, D.M. Jr. and R.A. Gaggioli, 2001, "Rational Design of Vehicles: I. Multi-Objective Optimization", ASME AES-Vol 41, ASME, New York

Paulus, D.M. Jr. and R.A. Gaggioli, 2001, "Rational Design of Vehicles: II. Subsystem Decomposition", ASME AES-Vol 41, ASME, New York

THE EXERGY OF LIFT, AND AIRCRAFT EXERGY FLOW DIAGRAMS

David M. Paulus, Jr. and Richard A. Gaggioli
Marquette University
Milwaukee, WI

ABSTRACT

Aside from incidental, auxiliary loads, in level flight the principal load on the aircraft propulsion engine is the power required to provide the continuous lift. To construct an exergy flow diagram for an aircraft – for example, for the purpose of exergy costing – an expression is needed for the exergy delivered to (*and by*) the wings. That is, an expression is needed for the exergy of lift. In this paper an expression is developed for this quantity, applicable not only in level flight but in other modes of flight as well. Then, three exergy flow diagrams are presented for a light aircraft, operating under three conditions: climb, level economy cruise, and level maximum power. The expression for lift exergy and these exergy flow diagrams are relevant to the optimal design of aircraft subsystems [1].

NOMENCLATURE

Symbol	Description
a	Aspect ratio
A	Area
C_d	Coefficient of drag for an infinite span
C_D	Coefficient of drag for a finite span
C_l	Coefficient of lift for an infinite span
C_L	Coefficient of lift for a finite span
e	Oswald's efficiency factor
F	Force
g	Acceleration of gravity
P	Power
V	Velocity
X	Exergy
x	Specific exergy
Y	Altitude

Table 1: Symbols

Greek Symbol	Description
δ	Constant for determining Oswald's efficiency factor
η	Efficiency
ρ	Air density

Table 2: Greek Symbols

Subscript	Description
0	Dead state
δ	Destruction
x	x-component
y	y-component

Table 3: Subscripts

INTRODUCTION

The concept of exergy, a special case of Gibbs' available energy for a body and large medium, is a valuable tool in both the analysis, and the optimization of energy conversion systems. Its value in the optimization of energy systems lies in their decomposition, breaking a large system into subsystems and devices to be optimized separately [2], typically with the objective of minimizing cost.

Aircraft energy systems, however, have traditionally been optimized so as to minimize gross takeoff weight. Although this objective does reflect aircraft lifecycle cost as well as performance, it is not necessarily the objective variable of choice for optimization, nor does it lend itself to decomposition[1]. Recently there has been interest in applying second-law methodologies, including exergy analysis, thermoeconomics and entropy generation minimization [3], to aircraft energy systems[2].

Aircraft energy systems are unique, in that exergy is required not only to operate them but also to lift them and to hold them aloft. In order to complete an exergy analysis, or apply thermoeconomics, it is necessary to create exergy flow diagrams. Required for this task is an expression for the exergy of lift.

Traditionally, the exergy associated with the application of a force has been given as

$$\dot{X}_F = \mathbf{F} \cdot \mathbf{V} \tag{1}$$

This is in fact a special case; the general expression is actually

$$\dot{X}_F = \mathbf{F} \cdot \left(\mathbf{V} - \mathbf{V_0}\right) \tag{2}$$

It is only because the "dead state" velocity is typically taken to be zero that (1) is then correct. However, for aircraft, equation (1) incorrectly leads to a conclusion that the exergy transport associated with lift is zero in level flight; such a conclusion does not charge an on-board energy system for the exergy it consumes in "staying aloft".

[1] For a discussion of objective functions for vehicles, see Paulus and Gaggioli [1]

[2] For a general overview of second-law methodologies, see Bejan et al. [4]

Finding a proper V_{y0}, or dead state velocity in the vertical direction, allows a proper calculation of the exergy of lift.

FINDING THE "DEAD STATE" VELOCITY

McCormick [5] states that the minimum induced drag coefficient of a wing is

$$C_{Di,min} = {C_L^2}/{\pi a} \quad (3)$$

where a is the aspect ratio. For drag to be the minimum while producing a given amount of lift, no parasitic drag may be present, and the induced drag coefficient must be given by (3). This minimum drag force is given by

$$F_{drag,min} = \frac{\rho A V_x^2 C_L^2}{2\pi a} \quad (4)$$

The minimum exergy input to the wing, assuming $V_{x,0} = 0$ and steady flight, is therefore

$$\dot{X}_{thrust,min} = \frac{\rho A V_x^3 C_L^2}{2\pi a} \quad (5)$$

An exergy balance on this wing in level flight results in

$$0 = {dX}/{dt} = \dot{X}_{thrust,in} - \dot{X}_{lift} - \dot{X}_{\delta} \quad (6)$$

The exergy destruction for the minimum thrust exergy input must be zero. Using this, and with the substitution of[3] (2) for $\dot{X}_{lift}$ and (5) for $\dot{X}_{thrust,in}$ yields the following expression for level flight (where V_y=0 and the lift force equals $m_a g$, where m_a is the mass of the aircraft):

$$-m_a g V_{y,0} = \frac{\rho A V_x^3 C_L^2}{2\pi a} \quad (7)$$

The dead state velocity is then found to equal

$$V_{y,0} = -\frac{\rho A V_x^3 C_L^2}{2\pi a m g} = -\frac{2 m_a g}{\pi a \rho A V_x} \quad (8)$$

With (8), the expression for the steady state exergy of lift for any component of mass m_c is

$$\dot{X}_{lift} = m_c g\left(V_y + \frac{2 m_a g}{\pi a \rho A V_x}\right) \quad (9)$$

A NOTE ON THE DEAD STATE VELOCITY

The rate of climb in a light aircraft is well approximated [5] with

$$V_y = \frac{P - P_{req}}{mg} \quad (10)$$

[3] in the y-direction

where P is the power supplied, and P_{req} is the power required for level flight.

If the power supplied is zero, and the aircraft is ideal[4],

$$V_y = -\frac{\rho A V_x^3 C_L^2}{2\pi a m g} \quad (11)$$

This is noted to be the same as V_{y0} given by equation (8). Thus, at a given forward velocity V_x, the dead state velocity corresponds to the speed of descent of an ideal wing, at zero power input, that is to say, in a steady-state glide.

CREATION OF EXERGY FLOW DIAGRAMS

The relationship for the exergy of lift will presently be applied to a light aircraft, the Glastar™ homebuilt aircraft, developed by Arlington Aircraft Development, Inc. A Lycoming O-320 or O-360 engine producing between 112 and 149 kW (150 to 200 hp), turning either a constant-speed or fixed pitch propeller, typically powers this aircraft. The aircraft has a gross weight of 889 kg (1960 pounds). The Glastar has a NASA GAW-2 airfoil, a wingspan of (10.7 m) 35 feet, and an aspect ration of 9.6. For the purpose of this article, the aircraft will be assumed to be powered by a 119 kW (160 hp) O-320 engine with a constant-speed Hartzell propeller and to have an empty weight of 544 kg (1200 pounds).[5]

Three exergy flow diagrams will be presented, one for full-throttle cruise (75% power) at an altitude of 2440 m (8000 feet), one for 65% power cruise at 2440 m, and one for maximum rate of climb at sea level..

The aircraft was divided into the following components: Engine, propeller, alternator, wing, fuselage and empennage (the horizontal and vertical stabilizers), and cargo. Trim drag and non-wing parasitic drag were attributed to the fuselage and empennage component.

The Aircraft Flight Model

The aircraft performance model employed is vastly simplified. The wing performance was assumed to follow the equation [5]

$$C_D = C_{Di,\min} + \frac{C_L^2}{\pi a e} \quad (12)$$

where e, Oswald's efficiency factor is given by

$$e = \frac{1}{1 + \delta + k\pi a} \quad (13)$$

[4] Here, imagine an aircraft producing no parasitic or trim drag.

[5] As these aircraft are amateur-built from a basic airframe kit, each individual aircraft is unique. Variations in power plant and propeller selection, avionic selection, construction technique, painting and interior finish lead to widely varying empty weights.

The value δ was set to 0.17 based on McCormick Figure 4.21. The value k was found from a best fit to published C_d versus C_l data [6].

The parasitic drag of the remainder of the aircraft and the trim drag were lumped together and calculated via

$$F_{drag,fuse} = \frac{1}{2}\rho A_{equiv}V^2 \tag{14}$$

The equivalent area was estimated to be 0.3 m^2 (3.225 ft^2) by tuning the model to best approximate published aircraft performance.

Although the performance model is perfectly adequate for the purpose of this paper, it tends to underestimate the aircraft's climb performance.

Exergy Balances

The exergy flow diagrams were created by applying the following exergy balances.

Fuel:

$$dX_{fuel}/dt = m_{fuel}g\, dY/dt - \dot{m}_{fuel}x_{fuel,ch} = \dot{X}_{lift,fuel} - \dot{m}_{fuel}x_{fuel,ch} - \dot{X}_{\delta,fuel} \tag{15}$$

Engine:

$$dX_{engine}/dt = m_{engine}g\, dY/dt = \dot{m}_{fuel}x_{fuel,ch} + \dot{X}_{lift,engine} - \dot{X}_{shaft,prop} - \dot{X}_{shaft,alt} - \dot{X}_{\delta,engine} \tag{16}$$

Propeller:

$$dX_{prop}/dt = m_{prop}g\, dY/dt = \dot{X}_{lift,prop} + \dot{X}_{shaft,prop} - \dot{X}_{thrust} - \dot{X}_{\delta,prop} \tag{17}$$

Alternator:

$$dX_{alt}/dt = m_{alt}g\, dY/dt = \dot{X}_{lift,alt} + \dot{X}_{shaft,alt} - \dot{X}_{elect} - \dot{X}_{\delta,alt} \tag{18}$$

Wing:

$$dX_{wing}/dt = m_{wing}g\, dY/dt = \dot{X}_{thrust,wing} - \dot{X}_{lift,net} - \dot{X}_{\delta,prop} \tag{19}$$

Fuselage and Miscellaneous:

$$dX_{fuse}/dt = m_{fuse}g\, dY/dt = \dot{X}_{lift,fuse} + \dot{X}_{thrust,fuse} - \dot{X}_{\delta,fuse} \tag{20}$$

Cargo:

$$dX_{cargo}/dt = m_{cargo}g\, dY/dt = \dot{X}_{lift,cargo} - \dot{X}_{\delta,cargo} \tag{21}$$

Other Relations

As stated previously, the exergy of lift provided to each component is proportionate to the weight of each component. For example, the exergy of lift provided to the engine was found with

$$\dot{X}_{lift,engine} = \frac{m_{engine}g}{m_{gross}g}\dot{X}_{lift} \tag{22}$$

Likewise, the net exergy of lift delivered by the wing was defined as

$$\dot{X}_{lift,net} = \dot{X}_{lift} - \frac{m_{wing}g}{m_{gross}g}\dot{X}_{lift} \tag{23}$$

Exergy flows from the propeller and alternator were found with the following two equations, respectively.

$$\dot{X}_{thrust} = \eta_{prop}\dot{X}_{shaft,prop} \tag{24}$$

$$\dot{X}_{elect} = \eta_{alt}\dot{X}_{shaft,alt} \tag{25}$$

Engine performance was taken from the Lycoming operator's handbook, and propeller efficiency was found via Hartzell's performance software. Alternator efficiency was assumed constant at 80%.

EXAMPLE EXERGY FLOW DIAGRAMS

The three example exergy flow diagrams are given below in Figures 1-3 The fuel is the source of exergy for the aircraft. However, note that the fuel itself requires a lift exergy input in the course of being kept aloft. The fuel's exergy is supplied, naturally to the engine, which converts it to mechanical power.

The engine's mechanical power is used to drive the propeller, and also auxiliaries, such as the alternator. The propeller creates thrust exergy, supplied to the wing to produce lift and to overcome the drag of the remainder of the aircraft.

The wing's exergy of lift is supplied to itself and all other components of the airplane Thus, a portion of the fuel's exergy makes a full circle journey in being converted to lift to keep itself in the air.

For cruise flight, the difference between inflows and outflows of exergy is the rate of exergy destruction in each component, as the diagrams are at steady state. For climbing flight, there is a rate of change in the component's exergy due to its increase in altitude.

Thus, Figures 1-3 show quantitatively the rates of exergy flow through the aircraft and the rates of destruction within each component.

CONCLUSIONS

If it is desired to optimize aircraft energy systems using the methods of thermoeconomics, an expression for the exergy of lift is needed. This is because a significant amount of exergy is required to hold the systems aloft. A suitable expression has been derived here, and employed to develop exergy flow diagrams. These diagrams, when combined with money balances, allow costs to be associated with the exergy flows. These costs are key to thermoeconomic decomposition. (See Reference [1].)

The diagrams here are based upon a very simple breakdown of the aircraft. A more detailed aircraft performance model could further break down exergy flows in the "fuselage and miscellaneous" category.

One might ask the question, "What happens to this lift exergy supplied to various aircraft components?" It is, of course, ultimately destroyed in the process of holding the system out of equilibrium with the environment. This is much the same as in other processes, such as heating a house. To keep a house at a higher (or lower) temperature than its environment, one must continuously supply heat exergy. This exergy is continuously "used up". Furthermore, just as one can "optimize" a house and its heating systems, such as by adding insulation or with a more efficient heating system, one can "optimize" an aircraft, by reducing weight and matching wings to the mission.

The advancements in aerospace technology suggest future work along lines similar to the foregoing. For example, at high speeds the proper "dead state" temperature might not be the temperature of the air surrounding the aircraft. Constraints to heat transfer with that air might require the dead state temperature to be taken as the stagnation temperature of the free stream. Additionally, the possible employment of plasma technologies in future hypersonic aircraft suggests a need for appropriate property relations of the exergy of plasma.

ACKNOWLEDGEMENT

This work was carried out under a contract with the Air Force Office of Scientific Research.

REFERENCES

1. Paulus, D.M. Jr. and R.A. Gaggioli, 2000, "Rational Objective Functions for Vehicles", AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, 8th, Long Beach, CA, Paper No. AIAA 2000-4182

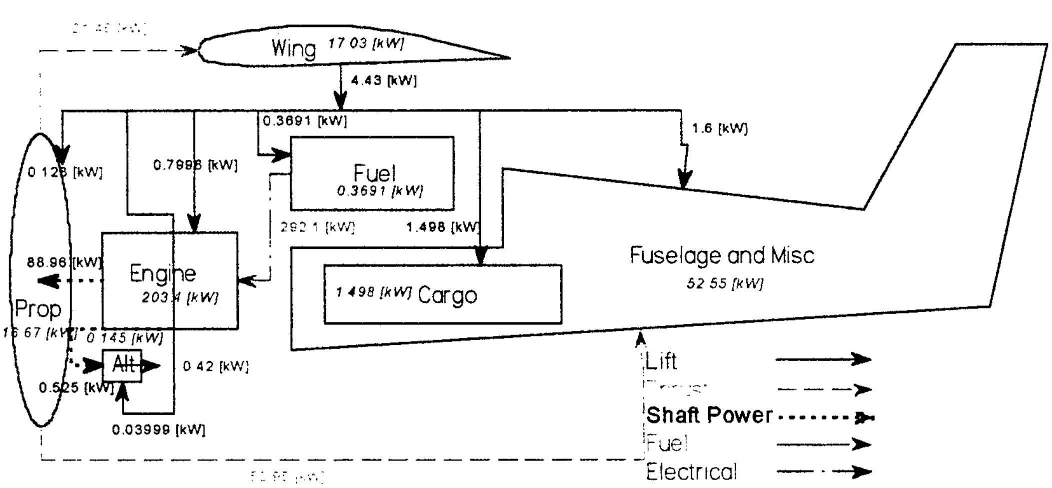
2. El-Sayed, Y.M. and Evans, R.B., 1970, *ASME Journal of Engineering for Power*, Vol. 92, p.27

3. Bejan, Adrian, 1999, "Role for exergy analysis and optimization in aircraft energy-system design", American Society of Mechanical Engineers, Advanced Energy Systems Division (Publication) Aes. v 39 1999. p 209-217

4. Bejan, Adrian, George Tsatsaronis and Michael Moran, 1996, *Thermal Design and Optimization*, John Wiley and Sons, NY

5. McCormick, Barnes W., 1995, *Aerodynamics. Aeronautics and Flight Mechanics*, John Wiley and Sons, New York, NY

6. McGhee, Robert J., W.D. Beasley and D.M. Somers, 1977, *Low-Speed Characteristics of a 13-Percent-Thick Airfoil Section Designed for General Aviation Applications*, NASA Technical Memorandum TM X-72697



Exergy destructions are shown inside of boxes.

Figure 1: Aircraft Exergy Flow, 75% Power, 8000 Feet

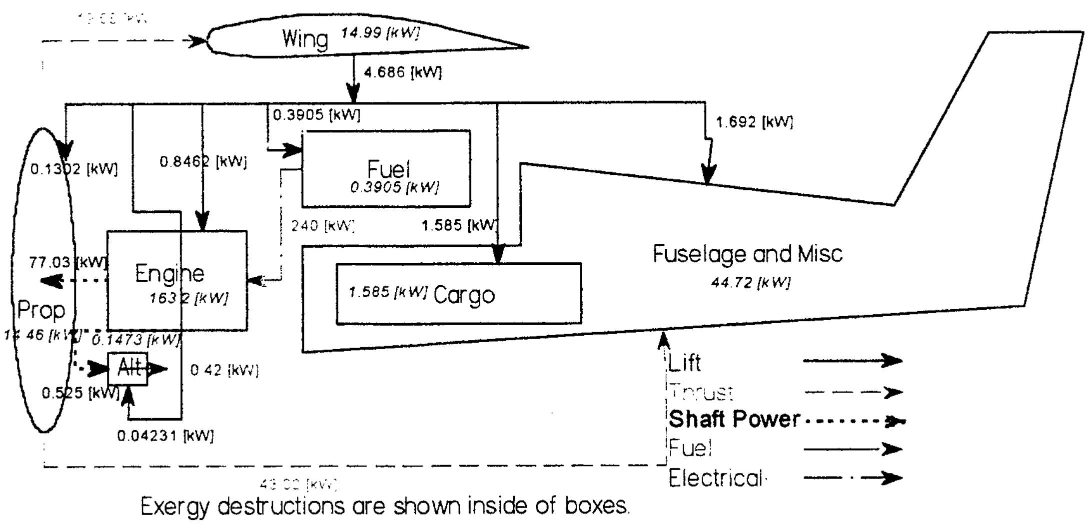


Figure 2: Aircraft Exergy Flow, 65% Power, 8000 Feet

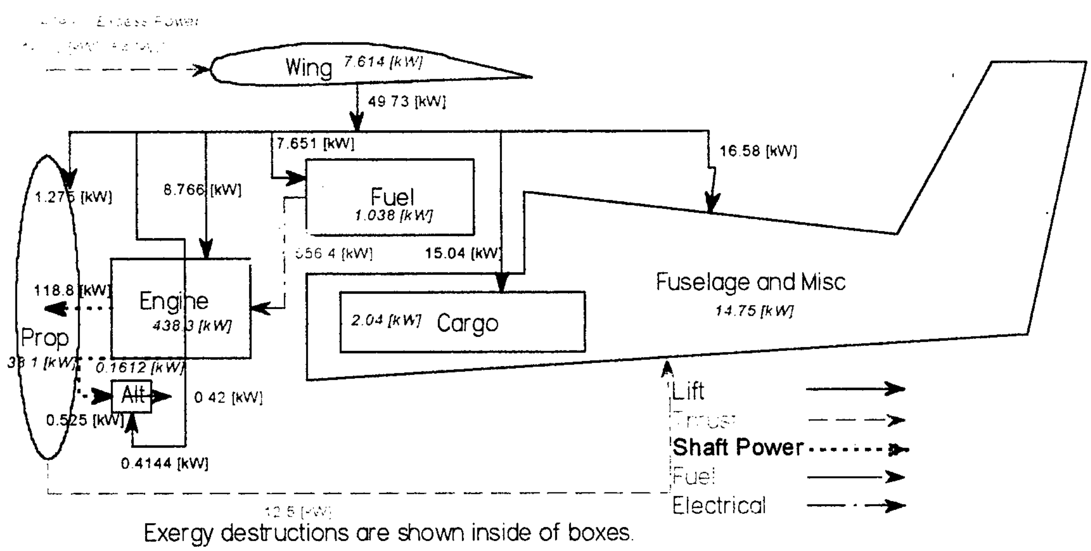


Figure 3: Aircraft Exergy Flows, Maximum Rate of Climb (1220 FPM), Sea Level

Rational Design of Vehicles
I. Multi-objective Optimization

David M. Paulus, Jr. and Richard A. Gaggioli
Marquette University

ABSTRACT

The customer for a vehicle typically has several desiderata, such as top speed, fuel economy, range, acceleration, ... Generally, these desiderata are conflicting. So, in order to deduce a single objective function, a means is needed for weighting (implicitly if not explicitly) the relative importance of these desiderata. That is, for weighting these "multiple objectives." This paper presents a rational methodology for developing a single-objective function to be optimized during the design of a vehicle. The methodology does require answers from the customer(s) to a straightforward set of questions, referring to the desiderata. Based on the answers, the objective function follows, mathematically, in a straightforward manner. An application to a light, personal aircraft serves as a case study.

INTRODUCTION

This series of papers is an expansion and clarification of material originally published in [1].

The customer for a vehicle typically has several desiderata, such as top speed, fuel economy, range, acceleration, ... Generally, these desiderata are conflicting. Consider this example: the customer of a light, personal aircraft might want a high cruising speed, good rate of climb, long range and a large payload. Clearly, these desiderata may conflict with each other. They all conflict with yet another desideratum – the minimization of expenditure.

If an optimal design for a vehicle is desired, it is desirable to define a single, "overall" objective from these multiple, "subsidiary" objectives, referred to here as desiderata[1].

The only means by which this can be achieved is to determine (implicitly if not explicitly) the relative importance of the individual desiderata. This requires that a common measure be found for the overall objective and the importance of each desideratum, and that a function be determined that relates the value of the design to the desiderata.

The common measure may be one of the desiderata, preferable one with which *all* of the other desiderata conflict, typically cost. The cost may be monetary, as would make sense for the light aircraft example, or it may be the number of total aircraft to be produced, a measure which would be perhaps of more utility for a military aircraft.

[1] Some will argue that there are better alternatives than defining a single objective. We will address that issue in our conclusions.

This paper introduces an overall objective function and a means of determining its component functions.

THE FUNCTION

The function that relates the desiderata to the value of the design will be referred to here as $V(D_1, D_2, \ldots)$, where the D_i's measure the various desiderata. The function to be optimized takes the form of a "profit" to be maximized,

$$\Pi = V(D_i) - Z - c_1F_1 - c_2F_2 - \ldots \qquad (1)$$

where Z is the capital invested (e.g., in monetary units or sacrificed production) the c's are the unit costs of the feeds, the F's in the previous formula. The obvious feed is fuel, but other feeds may be included, such as maintenance costs over the lifecycle of a vehicle.

DETERMINING THE FUNTION V

Clearly, before one can apply equation (1), one must find the function $V(D_i)$. One would expect the function to return zero for a very low level of performance for a given desideratum. That is, the level of performance is unacceptable.

As an example of this, consider an automobile. A person residing in the United States (with typical speed limits in the 100-125 km/hr range) might be unwilling to consider purchasing a vehicle incapable of at least 125 km/hr.

However, this person would likely be willing to pay more for an automobile that could achieve a higher speed than this minimal amount. Speed limits (or more correctly, the enforcement thereof) cause a reduction in the marginal value of a 1 km/hr increase in speed as the top speed of the car becomes higher and higher. This hypothetical person, therefore, might not be willing to pay any more money for an automobile capable of reaching 240 km/hr than an automobile capable of reaching 200 km/hr. (Acceleration would be a separate desideratum.) It is not rational to invest further resources in performance beyond the 200 km/hr "maximum" value."

Conversely, a particular automobile customer might state that no vehicle with a top speed below a "threshold value" of 140 km/hr would be acceptable.

Recognizing that the functions have a threshold value and some maximum value, one would expect the function $V(D_i)$ to return a value of zero up to a certain value of D_i, and become flat after a certain

value for that desideratum is reached. In between these points (the minimum acceptable and the maximum useful) lies some continuous function

A method is laid out below to estimate the function. It consists of four steps. It should be noted that in order, ultimately, to optimize a *subsystem*, it is absolutely necessary to develop this information, that is, the function $V(D_i)$, in some form, even if it is not done with the method used in this paper. The approach taken in this paper is, perhaps, the simplest conceivable. If a designer determines that the linear function is inappropriate for a given application, the method here would need to be extended.

First, it is assumed that $V(D_1, D_2, \ldots)$ can be reasonably well represented by a sum of functions, each of one variable:

$$V(D_1, D_2, \ldots) = V_1(D_1) + V_2(D_2) + \cdots \qquad (2)$$

Then, the following four-step procedure is proposed for finding each function V_i.

Step 1: Determining Median Performance

The foregoing algebraic "tradeoff functions" $V(D_i)$ for representing the value of performance may have an arbitrary shape between the points of minimum acceptable and maximum desirable performance. However, if a limited range of performance is considered, the assumption of a linear relationship is reasonable. One way in which this linear function may be constructed is around a median point. If a linear function is not satisfactory, this information (minimum acceptable, maximum desirable and standard performance) will still be of use in the construction of the function.

For a military combat aircraft, one way to find a median point is by considering the performance of the aircraft's adversaries, both current and projected. For each desideratum, at least one of the aircraft in the adversarial group has a best value. That value could be selected as the median of acceptable performance levels. The set of medians would form a "standard" of comparison for further investigations.

The same basic idea is applicable to civilian vehicles. Alternatively, the median values of a market segment might instead be chosen to set the "standard values."

Step 2: Projected Units Costs and Projected Production

A realistic estimate of both the cost per vehicle and the total production quantity of the vehicle should be made. This step, and the following, are particularly necessary if using production figures as a common currency.

Step 3: Projected Research, Design and Development costs

The projected research, design and development costs should be listed. When employed with the information attained above, the total project cost may be estimated.

Step 4: Algebraic Tradeoff Functions

The algebraic tradeoff functions, the $V_i(D_i)$, for each desideratum must be determined through questioning of the customers or end-users. As a bare minimum, the following three questions should be asked (in some form).

a. What is the minimum acceptable value for each of the desiderata?
b. Is there a point beyond which further improvement is not necessary?
c. How much would a given improvement, over and above the standard value determined in Step 1, be valued?

With the answers to these three questions determined, the simplest tradeoff function, linear, may be determined. Once again visiting the automobile example, let us imagine that the marketing department has asked these three questions to potential customers regarding the top speed of an automobile of a certain class. Suppose the average (or weighted average) answers were, S=125 km/hr as a minimum acceptable top speed, S=200 km/hr as a ceiling beyond which improvement has little or no value, and a willingness to pay 800 dollars for an improvement ΔS=10 km/hr over a standard 162 km/hr top speed. One can imagine a function, $V(S)$, which would have a value of zero dollars up to 125 km/hr and rise with a slope of 80 dollars/(km/hr) to a maximum of 6000 dollars.

However, for many cases the questions may not be best asked in terms of dollars. For a military vehicle, say an air superiority fighter, dollars would be a poor choice of units. In this example, the end-user (the Department of the Air Force or Navy) is not the same as the purchaser (Congress). The purchase costs have reached such high amounts that it is difficult for the average person to comprehend the sums in rational terms. Furthermore, neither body (Congress or the end-user) is spending their own money.

In such a case the questions may be rephrased in terms of production sacrifices; Question Number 3 could be changed to: "What reduction, in number of aircraft delivered to you, would you accept in order to obtain a specified improvement from the median value determined in Step 1?" A military leader should have a good grasp of tradeoffs between quantity and quality. The information from Steps 2 and 3 allows production tradeoffs to be converted to a dollar amount (or a cost to a production adjustment).

If a computer simulation were to be available that would predict aircraft survivability as a function of measured desiderata, it could be used to develop, or help develop, these trade-off functions.

The trade-off functions developed in this manner may be used directly in equation (1). If they are, however, equation (1) will normally result in a negative value. These trade-off curves do not take into account the base value of a vehicle performing at the standard levels. Nonetheless, the resulting function will be adequate for the purpose of comparing competing designs, as the standard values cancel in the comparison.

CASE STUDY: THE FUNCTION V FOR A LIGHT AIRCRAFT

The algebraic tradeoff functions were constructed for a personal, light aircraft as part of a case study. As this aircraft was to be an amateur-built aircraft, Steps 1-3 lack the importance they would have for a production run of vehicles, and are not considered here.

A User Questionnaire

In order to construct the objective function it was necessary to question the user about the value of different desiderata, in particular three.

It was decided, based on the light aircraft's typical "mission" profile, to consider speed at full power cruise at 8000 ft. (This works out to approximately 75% engine power for a normally aspirated piston engine.) as one desideratum, range at economy cruise (65% rated engine power) at 8000 ft as another, and finally and climb rate at sea level.

The following questions were posed to the end-user, and the following answers received.

1. When you were selecting an aircraft kit, what was the range of advertised cruise speeds of the aircraft (full throttle, 8000 ft.) in which you were interested? *Answer: 120 to 200 mph*
2. Approximately how much additional money would you be willing to spend for the aircraft in order to increase the cruise speed of your airplane by 5 mph above the average of the figures in question 1? *Answer: $350*
3. When you were selecting an aircraft kit, what was the range of advertised ranges of the aircraft in which you were interested (at 65% rated power, 8000 ft.)? *Answer: 500 to 1000 miles*
4. Approximately how much money would you be willing to spend in order to increase the range of your airplane by 50 miles over the average of the figures in question 3? *Answer: $300*
5. When you were selecting an aircraft kit, what was the range of advertised climb rates of the aircraft in which you were interested? *Answer: 700 to 1800 fpm*
6. Approximately how much money would you be willing to spend in order to increase the climb rate of your airplane by 100 fpm above the average of the figures in question 1? *Answer: $250*

The Overall Objective Function

With this information linear functions for the three desiderata were created. The linear functions for each desideratum are:

$$V_{V_C} = \$0 \text{ when } V_{cruise} < 120\text{mph}$$

$$V_{V_c} = \left(V_{cruise} - 120 \text{ mph}\right)\frac{70\$}{\text{mph}} \quad \text{when } 120 \text{ mph} \le V_{cruise} \le 200\text{mph} \tag{3}$$

$$V_{V_c} = \$5600 \text{ when } V_{cruise} > 200 \text{ mph}$$

$$V_{RC} = \$0 \text{ dol when RC<700 fpm}$$

$$V_{RC} = \left(RC - 700\text{mph}\right)\frac{\$2.5}{\text{fpm}} \quad \text{when } 700 \text{ fpm} \le RC \le 1800 \text{ fpm} \tag{4}$$

$$V_{RC} = \$2750 \text{ when RC>1800 fpm}$$

$$V_R = \$0 \text{ when } R\text{<500 miles}$$

$$V_R = \left(R - 500 \text{ mph}\right)\frac{\$6}{\text{mile}} \quad \text{when } 500 \text{ miles} \le R \le 1000 \text{ miles} \tag{5}$$

$$V_R = \$3000 \text{ when } R > 1000\text{miles}$$

Here V_C represents cruise speed at full throttle and 8000 ft., RC the rate of climb at seal level and R the range at economy cruise and 8000 ft.

Now the objective function, to be maximized, may be written as

$$J = V_{V_C} + V_{RC} + V_R - Z_{Capital} \tag{6}$$

It is desired to maximize the value of the aircraft's performance minus the cost of the aircraft. Maximization of J is equivalent to maximization of the profit P shown in Equation (1). They differ only by a constant, the hypothetical profit of the standard of comparison: $J = \Pi - \Pi_{standard}$

Application

The foregoing objective function was employed for two simple applications, relevant to a light personal aircraft. A kit had been purchased for construction of the aircraft, a Glastar™. The kit is not complete. The owner must complete the design of the airplane by selecting (or, in theory, designing) several component, such as an engine, an alterator, avionics...

The Glastar™ homebuilt aircraft, developed by Arlington Aircraft Development, Inc, is typically powered by a Lycoming O-320 or O-360 engine

producing between 150 and 200 hp, turning either a constant-speed or fixed pitch propeller. The aircraft has a gross weight of 1960 pounds. The Glastar has a NASA GAW-2 airfoil, a wingspan of 35 feet, and an aspect ration of 9.6. For the purpose of this article, the "standard" aircraft will be assumed to be powered by a 160 hp O-320 engine with a constant-speed Hartzell propeller and to have an empty weight of 1200 pounds.

For all calculations, it was assumed that the aircraft would have a lifespan of 20 years, be flown 200 hours per year, with 50% of those hours at economy cruise and the remaining 50% at high-speed cruise. Hours spent in climb are considered negligible. An interest rate of 8% was used.

Alternator Selection

First it was decided whether it is better to purchase a "standard" alternator or a lightweight model for the light aircraft. As no efficiency data is available, both alternators were assumed equally efficient. The standard alternator has an initial cost of \$294 and a mass of 5.9-kg (13-lbm). The alternative costs \$450, but has a mass of only 2.7-kg (6-lbm).

Performance of the aircraft was simulated with both alternators using a simple flight model.[2] The developed objective function predicts a decrease in present value profit of \$118 by choosing the lightweight alternator.

Engine Selection

Now, the objective function is used to choose between two possible engines. The 160-hp engine is the "standard" engine, chosen most widely by builders. The 180-hp engine will produce gains in full power cruise and climb rate, but at the expense of increased weight, decreased range, increased fuel consumption and increased capital cost. Table 1 shows predicted performance with both engines, estimated with the previous flight model.

Engine (hp)	Cost (k dollars)	Mass (kg)	Full Power Cruise (km/hr)/ Fuel Burn (kg/hr)	Fuel Burn @ 244 km/hr (kg/hr)	Rate of Climb (m/min)
160	22.3	116	259/22.9	18.8	372
180	26.2	122	271/29.9	21.8	442

Table 1: Performance with different engines

The objective function shows a \$10,800 decrease in present-value profit with the more powerful engine.

CONCLUSION

A rational methodology for developing a single-objective function for a vehicle has been presented here. It uses information from the customer, who is the ultimate judge of trade-offs in cost and performance, to develop this function. In Part 2, this information will be used, along with thermoeconomic methods, to optimally select vehicular energy conversion equipment.

It should be mentioned that two aspects of the foregoing development were, incidentally, simplistic and could be generalized. One was that the "user questionnaire" was addressed to only a single customer; this could easily be broadened by using modern, statistical market analysis methods. Also, the function $V(D_1, D_2, \ldots)$ was assumed to be a sum of single-variable functions of each D_i, and these functions were represented linearly. With a more elaborate battery of questions, higher order representations could be employed, including "coupling" terms involving more than one of the D_i.

Another point needs to be made, with reference to Footnote 1 presented earlier. Some might argue that it is not necessary to define a single overall objective. (Although we did not say that it was necessary, but "desirable.") Our value function $V(D_1, D_2, \ldots)$ is a means for weighting the relative importance of the different desiderata. In order to finally determine an optimum, the so-called "multi-objective optimization" methodologies (e.g., Fan and Shieh, 1983) must and do, ultimately, invoke weighting factors. Thereby, they establish the equivalent of a single objective. Concerns that use of a single-objective approach like ours could lead to finding a local optimum and not the global optimum, would be tantamount to suggesting that single objective functions should not be used even when there is only one objective. Moreover, in general it is easier to carry out an appropriate global search than to employ such techniques as the generation of tradeoff (Pareto) and/or indifference "curves," *especially* in those instances (usual) when there are more than two objectives – when the curves become surfaces and hypersurfaces. Furthermore, the value function V is determined by a rational procedure *before* the optimization process is begun, avoiding subjectivity that could enter when making "tradeoffs" toward the end of optimization procedures. Finally, we believe that with the approach advocated it is easier to keep the distinction clear between "desiderata" (desirable features of the product) and "objectives" (to be extremized).

ACKNOWLEDGEMENT

This work was carried out under a contract with the Air Force Office of Scientific Research.

REFERENCES

[1] Paulus, D.M. Jr. and R.A. Gaggioli, 2000, "Rational Objective Functions for Vehicles", AIAA-2000-4852, AIAA, New York

[2] See Paulus and Gaggioli [2].

[2] Paulus, D.M. Jr and R.A. Gaggioli, 2001, "Exergy of Lift and Aircraft Exergy Flow Diagrams", this volume

[3] Fan, L.T. and Shieh, J.H., 1983, "Multobjective Optimal Synthesis", *Efficiency and Costing*, ACS Symposium Series Vol. 235, pp. 307-332.

[4] Ostrofsky, Benjamin, 1977, *Design, Planning, and Development Methodology*, Prentice-Hall, Englewood Cliffs, NJ

[5] Woodson, Thomas T., 1966, *Introduction to Engineering Design, McGraw-Hill*, New York, NY

Rational Design of Vehicles
II. Subsystem Decomposition

David M. Paulus, Jr. and Richard A. Gaggioli
Marquette University

ABSTRACT

Overall vehicle design entails the subsidiary design – concurrent engineering – of the numerous subsystems that make up the vehicle. The engineer(s) in charge of the overall design must coordinate the work and integrate the designs of several "independent" subsystem design teams. That coordination and integration is achieved by specification of a particular set of constraints, imposed upon each design team. As the overall design proceeds, as a result of feedback from and review of the work of the individual teams, these constraints may be adjusted. Then many of the subsystems need to be re-designed. This procedure is carried out, iteratively, until "convergence" is achieved. That is, until the overall design is "suitable and satisfactory" in the judgment of the engineer in charge of the overall design, in consultation – directly or indirectly – with the "customer(s)." Better yet, the iterative procedure is continued until a rational objective function for the overall vehicle is optimized. In order to facilitate this concurrent engineering process, it is desirable that the design decisions, being made by the *subsystem* design teams, explicitly take into account the customers' weighted desiderata for the overall system. In this paper, it is shown how the methods of thermoeconomic decomposition can be applied (in conjunction with the rational objective function for the *overall* design presented in Part I) to deduce objective functions for optimizing the design of energy-conversion *subsystems*. An illustrative case study is presented, for the design-selection of an engine and a generator for a light aircraft.

INTRODUCTION

The design method to be presented here depends upon the initial development of a preliminary conceptual design.

Once a conceptual design has been accepted, the detailed design should proceed in an efficient fashion. It is not possible to optimize and design a vehicle, or even an energy subsystem, as an entirety. Vehicular energy systems are too complex to be designed by a single individual or team, as the number of decision variables becomes unmanageable. Therefore the design or selection of individual devices and/or subsystems is delegated to subordinate teams or individuals. It is desirable for these designers to have a methodology and information that allows them to make decisions in accordance with the overall goals of the vehicle. [1]

A vehicle typically relies on one fuel to achieve its goals. Because the subsystem or device design team is not optimizing a whole vehicle, but something that may produce or consume commodities not normally considered when looking at a whole vehicle, its objective function will vary from that of an overall vehicle. For example, the alternator on a light general aviation aircraft does not *directly* consume fuel, nor does it directly influence performance. Nonetheless, an aircraft with an alternator that is both lighter and more efficient, will, with all else remaining equal, perform better (and cost more). The person designing (or selecting) an alternator should not be burdened with, nor at this stage of the design be necessarily capable of, directly calculating the impact upon aircraft performance

The alternator mentioned above still has only one product (electrical energy). An onboard energy device or subsystem may use several "fuels" and/or supply several "products". These fuels and products, besides electricity and shaft power, include, but are not limited to, compressed air, hydraulic power and heat (or cooling). Additionally, for aircraft, one must account for the lift required to hold the device in the air (or to make it go up).

ASSUMPTIONS

The techniques presented here are for employment after a conceptual design has been penned. The (conceptual design) engineer(s) has estimated the weight, power and exergy consumption of the energy conversion systems. Therefore, only changes from the conceptual design are analyzed, as the "black boxes" are "filled in" in detail.

In this paper, when monetary values needed to be converted from present to time values, or vice-versa, it was assumed that the aircraft would have a lifespan of 20 years, be flown 200 hours per year, with 50% of those hours at economy cruise and the remaining 50% at high-speed cruise. Hours spent in climb are considered negligible. An interest rate of 8% was used.

THE DECOMPOSED OBJECTIVE FUNCTION

In general, a vehicular energy subsystem will fall into one of two categories. It may be an auxiliary. In the context of this paper, such a system is necessary

[1] Naturally, it must be determined which decision variables are local and which are global. That is, which variables a subsystem design team is free to vary in optimizing its individual subsystem, and which are constraints to the design team so as not to significantly affect the designs being carried out, simultaneously, by other design teams. These determinations may at times be made through "common sense", but at other times may require a sensitivity analysis.

(or desirable) for the operation of a vehicle, but the system does not *contribute* to any of the various desiderata, it can only detract, by adding weight to the vehicle or consuming exergy. Examples include components of the charging system and HVAC systems. Here such systems will be referred to *as non-propulsive subsystems*. Other on-board energy systems, notably components of the propulsion system, directly contribute to achieving the desiderata. The form of the objective function for a component will thus take one of two forms, dependant upon which function the component has.

The Objective Function for an On-board Non-propulsive Subsystem

An on-board subsystem is required, in general, to deliver a certain output as a design constraint. The goal in the design of such a subsystem is then to minimize its contribution to the total cost of the vehicle (which will maximize the "profit" of the vehicle; see Part I). The subsystem objective function, to be minimized, takes the form

$$J = Z + \sum_i W_i \left(\bar{c}_{m_i} m + \sum_j \bar{c}_{i,j} \dot{X}_{i,j} \right) \quad (1)$$

Here i represents each desideratum. W_i measures the relative importance of each desideratum. Average unit costs ($\bar{c}$'s) of either exergy or mass (subscript m) are evaluated with the vehicle operating in a mode where the individual desideratum is measured (e.g., full throttle at a specified altitude). The subscript j represents an exergy feed. For reasons described below, it is more convenient to associate a unit cost with mass than with lift exergy. (This is of course only relevant to aircraft.) Therefore the costs associated with lift exergy are accounted for with the mass cost, $\bar{c}_{m_i} m$.

From (1), an approximate expression for the change in the total cost of a vehicle due to a change in subsystem design may be written as

$$\Delta J = \Delta Z + \sum_i W_i \left(c_{m_i} \Delta m + \sum_j c_{i,j} \Delta \dot{X}_{i,j} \right) \quad (2)$$

This assumes relatively small changes in Z, m and $\dot{X}$. The c's now represent marginal exergy costs.

The Objective Function for a Propulsion Subsystem Component

While a non-propulsive subsystem typically has its output constrained, the propulsion system itself is a balance between the performance it supplies to the vehicle (e.g., as thrust to an aircraft) and its capital cost, weight and exergy consumption. Therefore, instead of only minimizing costs to achieve an optimal design, it is necessary to maximize its profit – the difference between performance delivered and the cost of providing it. The change in the overall vehicle's profit may be approximated with

$$\Pi = \sum_i W_i \left(\sum_j r_{i,j} \Delta \dot{X}_{i,j} - c_{m_i} \Delta m \right) - \Delta Z - \sum_i T_i \sum_k c_{i,k} \Delta \dot{X}_{i,k} \quad (3)$$

The j represents an exergy output, k an exergy input. Two new symbols appear in this function. The marginal unit revenue of an exergy output is represented with r. The subscript c distinguishes this components "profit" from the profit of the vehicle as a whole.

A time factor, T, is also now necessary. It represents the fraction of a vehicle's time spent operating in the regime where an individual desideratum is measured. The propulsion system component is only charged for its projected inputs. As an example, the top speed of an automobile may be quite important to the purchaser of a sports car, but the vehicle will spend little of its lifetime at full throttle, say 0.1%. This automobile will only see the high fuel flow associated with this operation 0.1% of the time.

The methods for finding the above-mentioned weighting factors and unit costs will now be explained. Creation of exergy flow diagrams for light aircraft is explained in detail by Paulus and Gaggioli [1].

DETERMINING THE WEIGHTING FACTORS

In Part I. of this series, an overall objective function was presented. For generic, global optimization of a vehicle, this is sufficient. However, for concurrent detailed design to take place, and equations (1) and (3) to be employed, weighting factors are needed.

The relative weighting factor for a desideratum D_i , as employed in this paper, is deduced from the algebraic tradeoff functions presented in Part I. It is given by

$$W_i = \frac{\dfrac{\Delta V_i / V}{\Delta D_i / D_i}}{\sum_k \dfrac{\Delta V_k / V}{\Delta V_k / V_k}} \quad (4)$$

Where V_i represents the value returned by the tradeoff function $V(D_i)$, D_i a performance desideratum, and ΔV_i the increase in price an end user would pay for a ΔD_i increase in performance. The numerator, then, represents the percent of expenditure increase the customer is willing to make, per percent increase in performance desideratum D_i. The denominator is the sum over all performance desiderata; so W_i represents the relative importance of D_i. V cancels, leaving

$$W_i = \frac{\Delta V_i \big/ \left(\Delta D_i / D_i\right)}{\sum_k \Delta V_k \big/ \left(\Delta D_k / D_k\right)} \tag{5}$$

DETERMINING THE UNIT COSTS

The proper unit exergy costs

As mentioned above, it is assumed that before detailed component design takes place, a "generic" design of the overall vehicle is present. Exergy and other unit costs to be used in Equations (1) and (2) for conceptual design are derived, at first, from the conceptual design. During detailed design of a component, the "filling in" of "black boxes", the component design engineer has no immediate effect on *equipment specifications* outside of his "box". (Obviously, the engineer's decisions will affect, to at least some extent, other equipment's performance. If need be, one can account for those effects in a second iteration of the overall component designs. And so on and so forth.) Therefore, the proper exergy costs (and values) to employ are *marginal* costs. In some cases these may be estimated by computing average unit exergy costs without consideration of capital costs. In other instances (e.g., where a jump to afterburning in a jet engine is necessary for increased thrust, or when neither a system's input nor its output is fixed), however, such methodology will result in error due to the average costs not approximating marginal costs [2].

Because the lift exergy requirements for an aircraft go up with the square of its weight, unit costs for lift exergy are an area where the use of average unit costs without consideration of capital costs will result in error. For this reason, marginal exergy costs for lift are instead accounted for in (1) and (3) as unit mass costs, derived below.

Fixing Marginal Costs and Revenues

If marginal exergy costs and/or revenues may be estimated by calculating average costs while neglecting capital costs, they are found by applying money balances to each component of the vehicle. The end goal of these money balances is to find a marginal unit cost and/or revenue of each exergy stream. The "source" of these costs is generally the vehicle's fuel, while the revenues result from vehicular performance.

While applying Equation (1) to non-propulsive subsystems, necessary auxiliary relations fixing a cost of an exergy stream the exergy costing for each desideratum D_i may be accomplished in one of two ways, depending on the nature of the desideratum.

Certain desiderata are measured at full engine power. Changes in subsystem exergy draw will not affect the overall fuel flow rate. It will, however, certainly affect the vehicle's performance. Therefore, for the case of desideratum D_i, unit exergy cost c_{ij} of a feed $\dot{X}_{ij}$ should be calculated by determining the marginal cost of the exergy flow which has the greatest direct impact on the desideratum. For example, this force is thrust exergy for an aircraft. This cost is found with

$$c_{i,j} = \frac{\partial \Pi}{\partial D_i}\frac{\partial D_i}{\partial \dot{X}_{i,j}} \tag{6}$$

Here D_i represents the performance desideratum and Π the overall vehicle profit. (See Part I of this paper.)

Other desiderata are measured at part throttle. Here, performance may be kept constant by varying engine power output while varying subsystem exergy draw. Then, the consequential exergy cost of the fuel may be assigned.

Changes in subsystem exergy draw effect not only speed, etc. but also range. Therefore, when range is the considered desideratum (D_i) not only must the direct cost of fuel be considered, but also the indirect cost of the increased fuel burn on the range R of the vehicle. This can be accomplished by adding the following term to the base cost of fuel.

$$c_{fuel,range} = \frac{\partial \Pi}{\partial R}\frac{\partial R}{\partial m_{fuel}} \tag{7}$$

Although this marginal cost is shown per unit mass, it could also be calculated on a per unit exergy basis.

Components in the propulsion or drivetrain system of a vehicle both consume exergy, and supply exergy that drive vehicle performance. Hence both terms for unit exergy cost and revenue appear in equation (3). In this equation, marginal costs (c's), may be determined from the known unit price of fuel (adding the cost to range if necessary) while revenues (r's) should be determined in the same manner as with equation (6), with

$$r_{ij} = \frac{\partial \Pi}{\partial D_i}\frac{\partial D_i}{\partial \dot{X}_{ij}} \tag{8}$$

The Marginal Cost of Mass

The marginal cost of mass, appearing in both Equations (1) and (2), may be found with

$$c_m = \frac{\partial \dot{X}_{lift}}{\partial m} c_{lift} \tag{9}$$

where $\dot{X}_L$ is the exergy of lift and c_L its unit cost.

With the exergy of lift devoted to a component of mass m_a given by [1]

$$\dot{X}_{lift} = m_c g\left(V_y + \frac{2m_a g}{\pi a \rho A V_x}\right) \tag{10}$$

equation (8) becomes, when applied to the whole aircraft,

$$c_m = \left(gV_y + \frac{4g^2 m_a}{\pi a \rho A V_x} \right) c_{lift} \qquad (11)$$

CASE STUDY: A LIGHT PERSONAL AIRCRAFT

These methods were applied to the Glastar aircraft described in Part I. to select optimally an on-board non-propulsive subsystem, and to optimally select an engine.

The Weighting and Time Factors

First, weighting factors are calculated for the three desiderata, described in Part I.: maximum cruise speed at 8000 ft., range at 65% rated engine power and 8000 ft. and maximum sea-level rate of climb. The tradeoff functions were given in Figures 1-3 of Part I. Using equation (4), the weighting factors were calculated to have the values in Table 1.

As stated in Part I, the aircraft was assumed to spend 50% of its flight time in maximum-speed cruise (full throttle, i.e., 75% rated power, at 8000 ft.) and 50% of its flight time in economy cruise (65% rated power at 8000 ft.). Time spent in climb was considered negligible. These assumptions yield time factors (T_i) of 0.5, 0.5 and 0.0.

Desideratum	Weighting Factor
Cruise Speed	0.565
Range	0.283
Climb Rate	0.153

Table 1: Weighting Factors

Exergy Costing

For the case of a light aircraft, it was assumed that marginal exergy costs could be approximated with average exergy costs, which were calculated from money balances while ignoring capital costs. When finding these costs, all lift exergy and all shaft power was assigned the same cost. In climb, stored exergy (due to the available energy of the aircraft and gravitational field) was assigned a value of zero. Fuel was assumed to have a cost of two dollars per gallon, a density of six pounds per gallon and have a chemical exergy content of 46,000 kJ/kg.

Balances The following seven money balances were used, one for each major airplane component (shown on the exergy flow diagrams in a companion paper, "Exergy of Lift, and Aircraft Exergy Flow Diagrams", Paulus and Gaggioli, this volume).

Fuel:

$$\begin{aligned}&\hat{c}_{fuel}\dot{m}_{fuel} + \overline{c}_{lift}\dot{X}_{lift,fuel}\\ &-\overline{c}_{fuel,total}\dot{X}_{fuel} = 0\end{aligned} \qquad (12)$$

Engine:

$$\begin{aligned}&\overline{c}_{fuel,total}\dot{X}_{fuel,ch} + \overline{c}_{lift}\dot{X}_{lift,engine}\\ &-\overline{c}_{shaft}\left(\dot{X}_{shaft,prop} + \dot{X}_{shaft,alt}\right) = 0\end{aligned} \qquad (13)$$

Propeller:

$$\begin{aligned}&\overline{c}_{shaft}\dot{X}_{shaft,prop} + \overline{c}_{lift}\dot{X}_{lift,prop}\\ &-\overline{c}_{thrust}\dot{X}_{thrust} = 0\end{aligned} \qquad (14)$$

Alternator:

$$\begin{aligned}&\overline{c}_{shaft}\dot{X}_{shaft,alt} + \overline{c}_{lift}\dot{X}_{lift,alt}\\ &-\overline{c}_{elect}\dot{X}_{elect} = 0\end{aligned} \qquad (15)$$

Wing:

$$\overline{c}_{thrust}\dot{X}_{thrust,wing} - \overline{c}_{lift}\dot{X}_{lift,net} = 0 \qquad (16)$$

Fuselage and Miscellaneous:

$$\overline{c}_{thrust}\dot{X}_{thrust,misc} + \overline{c}_{lift}\dot{X}_{lift,misc} = \dot{C}_{misc} \qquad (17)$$

Cargo:

$$\overline{c}_{lift}\dot{X}_{lift,cargo} = \dot{C}_{cargo} \qquad (18)$$

The $\dot{C}$'s are the rate at which money is flowing to a component to keep it aloft with the rest of the aircraft. That is, for the cargo, the cost of transporting it and for the fuselage, the cost of having a place to put the cargo, etc.

For each of the modes of operation (full power cruise, economy cruise and climb) the exergy flow rates were taken from the exergy flow diagrams in the companion paper. Then (for each mode) there are eight quantities, c's and $\dot{C}$'s, to be determined with the help of these seven equations, (11)-(17). One of the eight needs to be and can be assigned rationally, for each mode.

Assigned Unit Costs During maximum speed cruise and sea level climb, the engine is operating at full throttle. Therefore, a cost of thrust exergy must be found with equation (6), using the aircraft flight model (see [1]) and the algebraic trade-off functions shown in Figures 1 and 2 in Part I. The partial derivatives of cruise speed and climb rate with respect to thrust exergy were found to be 0.381 m/s-kW and 0.115 m/s-kW respectively. This, combined with marginal profit

for speed of 2.22 10^{-5} $\frac{\$/_{s}}{m/_{s}}$ from Figure 1 of Part I and a marginal profit of climb rate of 5.57 10^{-5} $\frac{\$/_{s}}{m/_{s}}$ from Figure 2 of Part I yielded unit exergy costs of thrust of 8.43 10^{-6} \$/kJ for cruise and 6.39 10^{-6} \$/kJ for climb.

Range is evaluated at 65% rated power at 8000 feet, a part throttle condition. The range of a propeller driven aircraft is well approximated with (McCormick, 1995)

$$R = \frac{\eta_{prop}}{\dot{m}_{fuel} \Big/ \dot{X}_{shaft} \frac{F_{drag}}{F_{lift}}} \ln\left(1 + \frac{m_{fuel}}{m_{empty}}\right) \quad (19)$$

Applying equation (7) to equation (19), using Figure 3 from Part I to evaluate $\frac{\partial \Pi}{\partial R}$ yields an added cost to fuel (for decreasing the range of the aircraft) of 0.414 \$/gal.

Exergy Costs Equations (12) through (18) were solved simultaneously using the unit thrust exergy or fuel costs given above to approximate the marginal unit exergy costs for full throttle cruise, climb and economy cruise.

Solving these equations simultaneously does not result in a good approximation for the marginal cost of fuel in a full-throttle condition. For the full-throttle cases, the marginal cost of fuel (exergy basis) is found from the price of fuel.

The approximate marginal exergy costs are given in Table 2.

	Full-Throttle Cruise	Climb	Economy Cruise
Lift	4.26E-5	2.01E-6	3.39E-4
Thrust	8.43E-6	6.94E-6	8.06E-5
Shaft Power	6.80E-6	4.71E-6	6.51E-5
Fuel	7.24E-6	7.24E-6	1.98E-5
Electric	1.09E-4	7.86E-6	2.14E-4

Table 2: Approximate Marginal Exergy Costs for the Light Aircraft, \$/kJ

Alternator Selection

Here it will be decided whether it is better to purchase a "standard" alternator or a lightweight model for the light aircraft. As no efficiency data is available, both will be assumed equally efficient. The standard alternator has an initial cost of \$294 and a mass of 5.9-kg (13-lbm). The alternative costs \$450, but has a mass of only 2.7-kg (6-lbm).

To make this decision, Equation (1), for a non-propulsive subsystem, is used. Terms for shaft power are unnecessary, as the power inputs to both units are the same. The cost of switching from the standard to the lightweight alternator will be given by

$$\Delta J = Z_{lightweight} - Z_{standard} + \sum_i W_i c_{mi} \left(m_{lightweight} - m_{standard}\right) \quad (20)$$

The individual terms have been given in tables 1 and 2. Equation (20) predicts that the lightweight alternator will have a present value *cost* of \$118 more than the standard. The standard alternator is the better choice.

This decision could be made by alternate means, directly employing the aircraft flight model and using equation 2, as was done in Part I. This method predicted an added cost of \$121. The \$118 value computed using the decomposed optimization of the alternator compares favorably to this \$121 value.

Engine Selection

A 160-hp engine is the "standard" engine, chosen most widely by builders. A 180-hp engine will produce gains in full power cruise and climb rate, but at the expense of increased weight, decreased range, increased fuel consumption and increased capital cost. (See Part I.).

Equation (3) for a propulsive subsystem may be employed in the form

$$\Delta J = \sum_i W_i \begin{pmatrix} \sum_{i,j} v_{shaft,i} \left(\dot{X}_{shaft,i,180} - \dot{X}_{shaft,i,160}\right) \\ -c_{mi} \left(m_{180} - m_{160}\right) \end{pmatrix} - \left(Z_{180} - Z_{160}\right) - \sum_i T_i \sum_i c_{fuel_i} \dot{m}_{fuel_i} \quad (21)$$

The unit values of power (i.e., exergy) may be taken from the exergy cost table, Table 2 for full-throttle cruise and the rate of climb, as the calculation methods are identical. For the part throttle flight at economy cruise, additional power capacity has no value, as speed, and therefore power output, is held constant. The weighting factors come from Table 1. The time factors, as stated previously, are 0.5 for full power cruise, 0.5 for economy cruise and 0 for climb. Fuel costs are calculated on a per unit mass basis here, as fuel is the raw energy source. The value used is 0.735 dollars/kg. (Exergy costing with the fuel exergy costs assigned would yield the same values.) The cost of fuel burn to range must be added to the cost of fuel for evaluation of range at economy cruise, as per equation (7), increasing the per kilogram cost of fuel to 0.912 dollars.

Evaluation of equation (21) shows a decrease of present value profit of 11,400 dollars for the 180-hp engine.

Alternatively, the method used in Part I directly employ the overall objective function. This results in decrease of profit (present value) of 10,800 dollars.

Although this differs from the 11,400-dollar decomposed value by 5.5%, the same decision would be reached in either case. The error likely results from the large shift from the preliminary design values,

shifts in engine power of 12%, (engine) weight of 6% and fuel consumption of 14%. In such cases iteration in the exergy costing step would improve results if necessary.

CONCLUSION

The combination of a rational objective function for a vehicle and the methods of thermoeconomics were used to decompose a vehicle into systems and subsystems in orders to make design decisions. These decisions were made with the presented objective functions for a non-propulsive or propulsive subsystem.

The results from these objective functions showed excellent agreement with the results of whole-vehicle simulation in combination with the overall objective function for a vehicle. The main conclusion, then, is that decomposition of a vehicle, for the purpose of effective concurrent engineering of subsystems, can evidently be achieved. In addition to exergetic costing, the key is the utilization of a rational objective function for the overall vehicle, to get the weighting factors needed for subsystem objective functions.

The next step in testing the methodology presented here is to apply it to subsystem design, instead of only subsystem selection. Then, the application to more complex vehicles could be pursued.

ACKNOWLEDGEMENT

This work was carried out under a contract with the Air Force Office of Scientific Research.

REFERENCES

[1] Paulus, D.M. Jr. and R.A. Gaggioli, 2001, "Exergy of Lift, and Aircraft Exergy Flow Diagrams", this volume

[2] Paulus, D.M. Jr., 2000, *Second Law Applications to Modeling, Design and Optimization*, Ph.D. Dissertation, Marquette University, Milwaukee, WI.

[3] Paulus, D.M. Jr. and R.A. Gaggioli 2000, "Rational Objective Functions for Vehicles", AIAA-2000-4852, AIAA, New York

[4] Paulus, D.M. Jr. and R.A. Gaggioli, 2001, "Rational Design of Vehicles I. Multi-objective Optimization", this volume